Fractional

COST Action CA15225

Fractional-order systems - analysis, synthesis and their importance for future design

Action book

edited by

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Thanks to International Science & Engineering Society, o.s. for the support while finalizing this book.

Competition, no matter in what area, is definitely something natural to all of us, but cooperation is another vital activity to move the society further.

It was the aim to cooperate that made the COST Action CA15225 “Fractional-order systems - analysis, synthesis and their importance for future design” to initiate and strengthen networking among research groups and individuals dealing fractional (almost) anything – covering math, systems design and their utilization to practical devices, although not being necessarily recognized by the users but taking advantage of it.

This book looks back the 66 months, the life-time of the COST Action CA15225 and is divided in two parts. The first summarizes the activities and events organized by the members to interact between organization, teams and even individuals and also advertise the fractional approach to a broader audience. The second part is represented by short papers giving an insight to research conducted by COST Action members contributing to specified Work Groups.

Thank you to all participants active during the COST Action life time for sharing their knowledge, experience and enthusiasm in research.

Jaroslav Koton
CA15225 Action Chair

Content

Part I

Shortly about the COST

COST (European Cooperation in Science and Technology) is a funding organization for research and innovation networks. These research and innovation networks are called COST Actions and help to connect research organizations, groups and individuals not just from Europe but also beyond. It is just about the members of the COST Action representing their country or institution how much they advertise the objective of the COST Action and take advantage from sharing their ideas in science and technology fields within the initial (introduced at the Kick-off Meeting) and hopefully during the life-time of the COST Action further expanding community.

Through COST Actions, different types of events and activities are organized (meetings, workshops, training schools, internships etc.), whereas the participants are supported from the COST Action budget. It is important to highlight that the research itself is basically not financially supported (at least not within the CA15225) from the budget of the COST Action. The budget is rather used to reimburse travel and accommodation expenses of active members serving as volunteers to COST Action willing to organize events, be tutors or trainers.

Every year COST runs a significant number of COST Actions. You may browse the current list and look for the opportunity to become member of the network linked to your field of research. If you do not find an appropriate COST Action, do the **big step** and submit your own proposal as we did.

For more reading about COST, please visit their web. There is all the information you need to start taking advantage of networking!

COST Action CA15225

Fractional-order systems have lately been attracting significant attention and gaining more acceptance as generalizations to classical integer-order systems. Mathematical basics of fractional-order calculus were laid nearly 300 years ago and since then have become established as deeply rooted mathematical concepts. Today, it is known that many real dynamic systems cannot be described by a system of simple differential equations of integer-order. In practice such systems are encountered in electronics, signal processing, thermodynamics, biology, medicine, control theory, etc. The COST Action CA15225 favors scientific advancement in the above mentioned areas by coordinating activities of academic research groups towards an efficient deployment of fractional theory to industry applications.

The COST Action CA15225 shows that the fractional calculus and its utilization is beneficial in different areas. The fractional-order systems, analogue or digital, used in control and regulation or specific signal processing may be seen as common, but mainly at the level of research community. The implementation of fractional approach to industry did not start running this COST Action but we believe that through the **Action Objectives**, **Deliverables** and other **Additional Outputs**, the CA15225 enabled to bridge separate research fields and disciplines to present interdisciplinary approach to scientific research and foster multidisciplinary breakthroughs beneficial to broader society.

Action Objectives

Although fractional calculus is primarily the domain of mathematicians, the COST Action CA15225 was understood to be multidisciplinary. Already by planning the COST Action, the activities were divided into Work Groups that although being individual expected the cooperation in between. As a result, four Work Groups were set to follow the **prime objective**:

Drive the European research and development in the field of description and implementation of fractional order systems in emerging fields of engineering and biomedical science, by overcoming the lack of common design and performance evaluation methods oriented on integer-order systems

and further contribute to both **research coordination objectives**:

- *Definition of optimization steps leading to efficient implementation of FO systems*
- *Improving characteristics of fractional-order controllers that can be employed in different industrial loops or in electro-mechanical systems*
- *Development of tools to define dynamic simulation models, control schemes and algorithms*
- *Design and implementation of FO controllers for industrial processes*
- *Design and characterize new fractional-order elements in order to obtain robust and commercial devices*
- *Utilization of fractional-order adjustment rule to model reference adaptive control in engineering applications*
- *Implementation of fractional-order digital/analogue function blocks especially in medical signal processing*
- *Penetration of the fractional-order models and systems in bioengineering and biomedical applications*
- *Characterization of properties preservation of non-integer order control and dynamical systems under discretization; new types of variational integrators*

and **capacity building objectives**:

- *Establishment of European-wide scientific and technology knowledge platform in order to instigate interdisciplinary interaction for the development of innovative fractional-order systems*
- *Bridging separate research fields and disciplines to present interdisciplinary approach to scientific research and foster multidisciplinary breakthroughs*
- *Ensure Early Career Investigators to participate in the Action within dedicated dissemination and formation activities such as workshops or STSMs and give them the best possible return in terms of scientific knowledge, research direction and coordination skills*
- *Increase the gender balance in terms of researchers involved in Action activities, both in terms of technical and scientific contribution as well as of research direction and Action governance*

Deliverables

Within each **Work Groups** (WG), the Tasks were defined and were solved by its members inside the Work Group or through efficient cooperation between the Work Groups to contribute to the defined Deliverables and also the overall Objectives of the COST Action. Although all Deliverables are understood to be delivered by the end of the COST Action CA15255, the individual topics are not closed and still offer possibilities for further improvements and progress, just being to be reached already out of this COST Action period. In total, 21 deliverables (Ds) were identified, linked to specific WG. For each deliverable, please see short summary.

Development and implementation of a general framework for global, non-adaptive identification of fractional and non-rational systems in general (WG1, D1)

A general framework for global, non-adaptive identification of fractional and non-rational systems in general has been developed in several specific directions. The first is non-parametric analysis, in which an overall shape of the frequency characteristic is analyzed first, and then a suitable explicit or implicit fractional order model is proposed and optimized by means of a PSO algorithm. Relevant study was reported in doi: 10.1016/j.ifacol.2017.08.2084 in collaboration between Serbian and Italian COST Action members. Parts of the research were also reported in doi: 10.1016/j.aeue.2017.05.036.

Next path under investigation is to develop specific forms of transfer functions related to various distributed-parameter systems, with or without fractional spatio-temporal dynamics. Initial results are reported in doi: 10.1109/ECCTD.2017.8093252, doi: 10.1007/s11071-016-3322-z. An important issue regarding stability of the developed fractional models, which also addresses issues of admissible values of parameters, is presented in doi: 10.1016/j.ifacol.2017.08.2091 and doi: 10.1049/iet-cta.2018.6350.

Finally, another technique has been developed where fractional models can be derived for time-varying systems based on frequency response functions obtained at the beginning and at the end of the transformation. This technique has been used to explain the variation in dynamic response of adhesives as they undergo phase transition during the curing process. For further information see doi: 10.1016/j.apm.2019.08.021.

Development and implementation **of a general framework of adaptive identification of fractional and non-rational systems** (WG1, D6)

A general framework for adaptive identification of fractional and non-rational systems in general has been developed. The research has targeted algorithms that are capable of identifying unknown parameters in transfer functions of arbitrary form, including rational, fractional, transfer functions, transfer functions with “fractional delay”, and other forms of transfer functions which are derived from partial differential equations describing distributed parameter systems. The proposed algorithm is gradient based, and novel convergence conditions are derived generalizing well known results regarding input richness. Relevant studies and approaches have been reported in doi.org/10.1016/j.aeue.2017.04.008, 10.1109/TAC.2019.2893973.

For practical performance of controller the design methods strongly depend on the relevancy of identified models. Therefore, the mathematical models should express meaningful dynamics of real-world systems. Two fundamental numerical solution methods of fractional calculus in identification and simulation problems of One Non-Integer Order Plus Time Delay with one pole (NOPTD-I) transfer function models were discussed and utilized. The identification process is carried out by estimating parameters of a NOPTD-I type transfer function template according to the experimental step response data. The reached results were conducted within the collaboration between Estonia (University of Tallinn) and Turkey (Inonu University) groups and the results were reported in doi: 10.1142/S1793962319410113. Part of this Deliverable is also the Matlab code, which is shared for the use of researchers in the Mathworks (https://www.mathworks.com/matlabcentral/fileexchange/88813-fractional-order-time-delay-plant-identification).

Approximation of derivatives and integrals **of fractional orders by new numerical and analytical methods; Fractionized models** (WG1, D13)

Numerous novel approaches for the numerical handling of fractional operators have been proposed recently by research groups within and outside of the COST action. In a detailed discussion of some of these methods doi: 10.3390/math8030324, a significant number of problems have been identified and improvements or alternatives that avoid these issues have been suggested. Furthermore, a new class of numerical methods has been proposed. These novel methods differ in their structure and in the interpretation of their respective parameters greatly from the traditional approaches. Therefore a comparison of the performance is difficult and appropriate concepts for such a sensible comparison need to be designed. This work is currently ongoing. We expect to have publishable results within the two years after the end of the action. The outcome will then be published in suitable international peer reviewed journals. Moreover, a first guide to the evaluation of fractional integrals and derivatives (according to different definitions) of the main elementary functions, has been provided, doi: 10.3390/math7050407, thus filling a gap in the scientific literature on the subject.

Regarding the fractionized models, their efficient applications in tumor-growth, tumor-immune surveillance and epidemiology were studied, see e.g. doi: 10.1063/1.5096159, doi: 10.11121/ijocta.01.2020.00862, or doi: 10.1016/j.chaos.2021.110654.

Development of methods for PID **controllers' synthesis, Matlab toolboxes and discretization algorithms** (WG1, D17)

Numerical methods for the solution of fractional differential equations, in particular for multi-order systems, i.e. systems in which each differential equation has a different order, and multi-term equations (when in the same equation there are several fractional derivatives) has been discussed, doi: 10.3390/math6020016 and doi: 10.3390/math8030324. As a result of this investigation a set of robust Matlab codes have been released. These are general purposes codes, with a similar usage to those of other built-in Matlab codes for classical ordinary differential equations, and their use is therefore very simple and possible also by users with no particular experience in numerical analysis.

Other results contributing to this deliverable deal with the investigation of multi-loop control structures. The multi-loop control structures can enhance inherent disturbance rejection performance of classical closed loop control loops. While the classical closed loop PID control loop (inner loop) deals with stability and set-point control, the additional model reference control loop of MIT rule (outer loop) can improve the disturbance rejection control performance without degrading the optimal set-point control performance. Such adaptive disturbance rejection approach, which is not influencing the set-point control performance, can be achieved by selecting reference models as transfer function of the PID control loops with ignorable time delay. This structure may deal with the design tradeoff between set-point and disturbance rejection performances for low time-delay systems. This approach is researched in a fruitful collaboration with Dr. Aleksei Tepljakov and Prof. Eduard Petlenkov from Tallinn University. Several variants of multi-loop Model Reference (Ml-MR) PID control designs (Ml-MR PID-MIT control) to improve disturbance rejection control were discussed doi: 10.3390/a13020038. Disturbance rejection control performance of multi-loop Model Reference FOPID control structures was reported in doi: 10.1142/S0218126618501761 and doi: 10.3390/a13080201. The Matlab code delivery was shared in the Mathworks (https://www.mathworks.com/matlabcentral/fileexchange/88823-multi-loop-model-reference-pid-control).

In recent papers, see doi: 10.1016/j.ejcon.2020.06.005 and doi: 10.23919/ECC.2019.8796163, classes of fractional-order PID controllers and distributed-order PID controllers were proposed and successfully applied to permanent magnet synchronous motors used in industry. The design of the controllers' parameters was made by a generalized particle swarm optimization, which was applied to the controllers in two nested loops of the electrical drive. Optimization was based on a cost function considering both performance and robustness indexes, i.e. the maximum sensitivity, maximum noise sensitivity, maximum resonant peak, while guaranteeing closed-loop stability. Results show that the proposed controllers can successfully replace usual PI/PID controllers both in reference tracking and disturbance rejection.

Moreover, other results regarded fractional-order control of robotic manipulators. In particular, in doi: 10.1109/SMC.2019.8914031 a general procedure was introduced to design fractional-order controllers for independent-joint control of DC motors actuating robot joints, for a 5DOF robotic manipulator. Both position and speed are controlled by employing feedback and feedforward actions. Design formulas provide the controllers' parameters as a function of frequency-domain specifications. A detailed simulation model allows to verify that better performance is achieved with respect to integer-order controllers, even in presence of disturbances and plant nonlinearities.

A paper doi: 10.1016/j.ifacol.2020.12.2050, investigated the use of a fractional-order lag network or a fractional-order PI controller for the motion control of three revolute joints of a manipulator. The introduced fractional compensators are designed by using the symmetrical optimum principle and by parameters optimization or by frequency-domain loop shaping, respectively. In both cases, the same performance and robustness specifications were considered. Simulation results and frequency response show effectiveness and robustness of the approach.

Finally, another activity (see doi: 10.2298/TAM201203016L) supporting this deliverable addressed the problem of finite-time stability for uncertain neutral nonhomogeneous fractional-order systems with time-varying delays. A robust finite-time stability test procedure based on the extended form of the generalized Grönwall inequality was suggested. The sufficient condition for robust finite-time stability of such systems was established. Numerical examples show the effectiveness of the procedure.

Presentation of optimized low-order approximations of fractional Laplacian operator featuring lower circuit complexity (WG2, D2)

The results presented in doi: 10.1016/j.ifacol.2017.08.1422 and doi: 10.1109/ECCTD.2017.8093324 provide efficient techniques to obtain accurate and low-order approximations of fractional operators and compensators that are useful for control and other applications.

Partially also delivered by presenting the comprehensive analysis of the approximation of low-pass magnitude response. These results were presented in doi: 10.1016/j.aeue.2017.04.031. The further analysis of specific transfer function types continues, see e.g. doi: 10.5755/j01.eie.24.2.20634A curve fitting based technique is introduced for approximating the behavior of fractional-order systems, and it is applied in the case of filters, controllers, and driving point impedances. The magnitude and phase frequency responses of the transfer function are first extracted and approximated through curve fitting-based techniques. A rational integer-order function is then obtained and realized using appropriately configured passive and/or active topologies. Comparison between the conventional method and the proposed method reveals that the achieved benefit is the significant reduction of the passive and/or active components count. The concept as well as the related applications have been published e.g. in doi: 10.1109/NILES50944.2020.9257936, doi: 10.1007/s00034-020-01514-7, doi: https://doi.org/10.3390/fractalfract4040054, or doi: 10.1016/j.aeue.2020.153537.

Presentation of tools for analysis and synthesis of the fractional-order **function blocks** (WG2, D7)

The FOMCON toolbox for MATLAB®, initially developed by Aleksei Teplakov, the MC member for Estonia, was further improved. The FOMCON toolbox for MATLAB® is a fractional-order calculus based toolbox for system modeling and control design. For more details about the Matlab Toolbox you may check the **MathWorks® File Exchange** or the **FOMCON homepage**.

During the COST Action period the FOMCONpy was introduced and is a new fractional-order modelling and control toolbox for Python. It is an extension of the existing FOMCON toolbox for MATLAB, but this time aiming at Python users and the Internet of Things (IoT) community. Just like the original toolbox, it offers a set of tools for researchers in the field of fractional-order control. Similarly as FOMCON, also FOMCONpy is available for the broad research community: **https://github.com/outstandn/fomcon**.

Increased approximation accuracy of fractional **Laplacian operator for analogue circuit design, discrete rational approximations** (WG2, D14)

One of the contributions to this deliverable is in the comprehensive analysis of the Oustaloup approximation and defining the equations to determine the initial parameters of this approximation technique to obtain a response that satisfies the designers' requirements of approximation error in magnitude and/or phase in a specific frequency range for the minimal possible order N of the approximation as presented in doi 10.1109/ICUMT.2018.8631227.

Other results relevant to this deliverable were published in doi: 10.1109/CoDIT.2019.8820521. A link was established between the Lagrange's continued fraction expansion (CFE) and two other CFEs introduced for approximating the fractional Laplacian operator s^{ν}, with $0 < \nu < 1$, and their discrete realizations. On their turn, the two novel CFEs are linked with each other. Zeros and poles of these new approximations alternate on the negative real half-axis of the s-plane (for analog realizations) and on the real segment inside the unit circle of the z-plane (for discrete realizations). Discrete approximations the fractional operator have poles and zeros enjoying a nice symmetrical distribution on the z-plane, namely with respect to the origin of the *z*-plane. These properties are obtained for any order of realization (i.e. degree of numerator and denominator of the approximation, or number of zeros and poles).

An indirect approach in two steps was proposed in doi: 10.1109/SMC.2019.8914260 to obtain discrete rational transfer functions (TFs) for implementing the fractional-order Tustin operator (FTO). The polynomial coefficients of the rational discrete TF approximation of the FTO are given by closed-form expressions. In this way, an easy computation is possible, which is a remarkable new feature. The proposed coefficients expressions are the basis for proving the zero-pole interlacing of the discrete FTO. The interlaced zero-pole pattern shows a symmetrical configuration on the *z*-plane.

Presentation of new fractional-order elements based on the IMPC, graphene and RC-EDP design approach (WG2, D16)

The hardware design of fractional-order elements based on the Resistive-Capacitive Elements with Distributed Parameters (RC-EDP) was performed by implementing thick-film technology based fractional-order elements. A software design tool was developed by prof. Ushakov and presented at doi: 10.1109/ECCTD.2017.8093314 and later further described in detail doi: 10.1016/j.jare.2020.06.021.

The limitations of assumed thick-film technology process were investigated and layout optimization recommendations described. Following these optimization steps the solid-state capacitive FOEs were produced and are available as utility samples; **capFOE_045** and **capFOE_05**.

Next to the investigation of solid-state FOE design based on the RC-EDP theory, with respect to the realization of analogue hardware devices, with intrinsic, fractional-order structure, another two different design technologies have been analyzed.

The first approach is based on the use of carbon-based structures dispersed inside a polymeric matrix. The article doi: 10.1109/TED.2020.2965432, analyzes the material characterization of the nanocomposite employed in the fabrication of a solid-state fractional capacitor. The studies on the nanocomposite characterization include the Fourier-transform infrared (FTIR) spectroscopy spectra, the Raman spectra, the X-ray powder diffraction (XRD) spectrum, the transmission electron microscopy (TEM), and the scanning electron microscopy (SEM) images, while in doi: 10.1016/j.mejo.2018.10.008, the possibility of realizing fractional capacitors by using carbon black nanostructured dielectrics was investigated. Capacitors have been realized by varying the percentage of distributed carbon black. The frequency analysis of the capacitors has been, therefore, performed. The Bode diagrams outline that this class of devices shows a non integer order behavior. Moreover, a dependance between the curing temperature and the fractional order has been shown.

In the next approach, doi: 10.1016/j.aeue.2019.152927, the team proposes and demonstrates the possibility of using Bacterial Cellulose (BC), a bio-derived polymer, for the realization of fractional-order electronic devices. BC, unlike plant-derived cellulose, is produced by some genera of bacteria, if a suitable culture is maintained. Compared to plant-derived cellulose, BC can be obtained with a green and low-energy production process, which does not produce pollutants nor carbon composites. BC is used as the bulk in a capacitor-like structure. The device impedance has been investigated and experimental evidence of its fractional nature is given. A model is proposed and a possible explanation of the involved phenomena is provided.

Development of integrators **and differentiators, initial implementation of digital fractional order blocks** (WG3, D3)

Efficient circuit solutions to design fractional-order integrator and differentiators using opams, CCIIs, CFOAs and OTAs were published e.g. in doi: 10.1109/TSP.2017.8076081. Further approximations of fractional-order differentiator and integrator operators are proposed in doi: 10.1002/cta.2598. These approximations target the realization of these operators using standard active filter transfer functions. Hence, circuit implementations in integrated circuit form or in discrete component form are significantly facilitated. The concept is based on the employment of the partial fraction expansion tool and, as a result, the fractional-order transfer function is decomposed in a sum of a constant term and 1st or 2nd basic filter functions (i.e. lowpass, highpass, bandpass etc). Comparison with the literature shows that a significant reduction of the required circuit complexity is achieved. Applications of this concept have been published in a number of papers: doi: 10.1016/j.mejo.2018.11.013, doi: 10.1016/j.mejo.2019.05.002, doi: 10.1007/s00034-019-01308-6, doi: 10.3390/technologies7040085, or doi: 10.3390/electronics9010063.

Controllable fractional-order integrator, integrational-derivative two-port and practical aspects of their mutual interconnection has also been investigated. The minimal configuration blocks of fractional-order immitances with only one active element and fractional-order integrator in current mode were designed. The reached results and proposed solutions were published in doi: 10.1109/TSP.2019.8768814, doi: 10.3390/app10010054, doi: 10.1109/TSP49548.2020.9163553 and doi: 10.1109/ICECS49266.2020.9294923.

Development of fractional-order analogue and digital filter topologies using basic building blocks (WG3, D9)

Analogue or digitally controlled analogue fractional-order filters providing the low-, band- and high-pass frequency response. Some of the basic results reached by Action members can be found e.g. in doi: 10.1016/j.aeue.2017.04.031, doi: 10.5755/j01.eie.24.2.20634, doi: 10.1109/TSP.2018.8441421, or doi: 10.1515/jee-2018-0001. During the design, the attention was paid to proposing circuit solutions featuring also optimized circuit complexity. This was partially reached by designing and experimental verification and optimization of RC structures with distributed parameters and by designing suitable values of fractional-order series in order to cover ranges required by circuit applications. Comprehensive research on the design of functional fractional-order blocks, their verification and optimization based on the target features was made. Moreover, analysis of fractional-order transfer functions of various filter types was delivered. Several reconfigurable and reconnection-less reconfigurable filtering structures have been also designed. Finally, we have studied building blocks suitable for applications in fractional-order domain. Results relevant to this deliverable and reached by COST Action members can be found in: doi: 10.5755/j01.eie.25.3.23673, doi: 10.1016/j.jare.2020.06.022, doi: 10.1109/TSP.2019.8769089, doi: 10.1109/TSP49548.2020.9163400.

Optimal dynamic control – methods for solution and numerical computation of optimal dynamic control (WG3, D11)

The activities within this deliverable mainly base on further development of the FOMCON toolbox that was originally developed by Aleksei Teplakov: https://de.mathworks.com/matlabcentral/fileexchange/66323-fomcon-toolbox-for-matlab. During the COST Action period the FOMCONpy was introduced and is a new fractional-order modelling and control toolbox for Python and is also available for the community for further usage and design of fractional controllers: https://github.com/outstandn/fomcon.

Application of FO linear and non-linear blocks in sensors, actuators and control systems (WG3, D18)

The activities on this deliverable were initiated by utilizing fractional-order function blocks in the PID control: see e.g. doi: 10.1142/S0218126618501761, doi: 10.1016/j.ifacol.2018.06.151, or do: 10.1109/TSP.2018.8441247.

Fractional-order PID controllers and fractional controllers with various distributed orders were further designed, realized, and applied to control electrical drives and robots, in several applications – see doi: 10.1016/j.ejcon.2020.06.005, doi: 10.2298/TAM201203016L, doi: 10.1109/JAS.2017.7510325, doi: 10.1007/978-3-030-17344-9_11, doi: 10.1109/SMC.2019.8914031, doi: 10.1016/j.ifacol.2018.06.154, whereas detail description of the PID controllers and their purpose and application is part of Deliverable D17.

Development of preliminary models for automotive injection systems and of physical and mathematical knowledge on models of Havriliak (WG4, D4)

Models describing individual parts that affect the injection process in advanced automotive natural gas engines were discussed in doi: 10.1016/j.ifacol.2017.08.2084. Gas engines were proposed to reduce pollution determined by combustion of Diesel or gasoline fuels, but their performance strongly depends on the metering of the air/fuel ratio, which is achieved by controlling the gas injection timing and the gas pressure in the common rail volume. The activities aimed to define an accurate model that would represent the gas pressure dynamics in the injection system. It was identified that a fractional-order model describes the gas flow into the common rail better than ARX integer-order models of high order. Identification was made in the frequency domain by minimizing a difference criterion between the model output and real data.

Moreover, the report "Fractional-Order Modeling of Fuel Propagation in Electro-injectors Pipes" by F. Saponaro, G. Maione, P. Lino, R. Garrappa, (Workshop on current progress in fractional-order systems and their utilization – Cost Action 15225 Annual Workshop, San Sebastian, Spain, 5-6 October 2017), showed results in modeling strategic components of common rail Diesel compression-ignition engines, namely the electro-injectors used to to let the proper amount and rate of fuel enter the combustion chamber. Preliminary fractional-order models of the fuel flow inside the electro-injectors were developed to better describe certain fluid-dynamic processes associated with the high-pressure flow according to wave propagation.

Preliminary mathematical developments and properties regarding Havriliak-Negami type of time-domain or frequency-domain models were obtained by putting together the best results from the following previous achievements doi: 10.1007/978-3-319-45474-0_38 and doi: 10.1515/fca-2016-0060 reached by the COST Action participants. Some advancements were proposed in the current output doi: 10.1515/fca-2020-0002. In details, numerical methods are now available to give an approximate but accurate solution to differential equations in which the Prabhakar derivative is used to better describe anomalous relaxation in Havriliak-Negami models of dielectric materials or biological tissues. Moreover, the time-domain relaxation and response functions of the most common materials that show anomalous relaxation are now well known and described.

Detailed models and virtual prototypes; low-order models for FO controllers, development of simulation tools for systems of Havriliak-Negami type (WG4, D8)

The results shown in the previous publications doi: 10.1016/j.ifacol.2016.08.071, doi: 10.1109/CDC.2015.7403160, doi: 10.3182/20150218-3-AU-30250//978-3-902823-71-70075, doi: 10.3182/20140824-6-ZA-1003.00889 are based on simulation models developed in the Matlab/Simulink environment or on virtual prototypes built by the AMESim software package. The mentioned tools also allow an easy and fast prototyping of the control systems on the basis of a model-based approach that guarantees accuracy and effectiveness of the results. Namely, the implemented simulation models allow an accurate representation of the complex, nonlinear, time-varying dynamical processes that occur in the considered common rail injection systems and characterize their operation in different working points. In this sense, the models can be considered very close to the hardware and real systems they represent. More specifically, the adoption of fractional order models for CNG injection systems provides a compact mathematical representation of the only most significant characteristics of the injection process. In fact, unlike the classical high integer-order ARX models, a simple model structure can be obtained in the form of a fractional-order transfer function by neglecting the secondary effects and capturing only the relevant features for control design, providing a reliable prediction of the injection pressure at the same time. The identification procedure is based on a frequency-domain method and on classical and efficient convex techniques applied to experimental data.

In doi: 10.3390/fractalfract4030037, a serial structure of cascaded, shifted, fractional-order, lead compensators was proposed as a new type of fractional-order controller. Two stages are connected in series and introduce their respective phase leads in shifted adjacent frequency ranges. The obtained compensator shows a nearly flat phase diagram in a large frequency range and can be easily realized by low-order rational transfer functions, each stage being a second-order transfer function with limited sensitivity of coefficients to parametric variations. However, the number of stages and their free parameters can be changed such that these new controllers can be flexible and suitable for solving difficult control problems. On this basis, a method is introduced to design a robust controller for a class of benchmark plants that are difficult to compensate due to monotonically increasing lags. The main design strategy consists in compensating the rapidly and monotonically increasing phase lag by the lead introduced by the compensator in the same frequency range. Desired set-point response, stability robustness to gain and parameter variations, and compensation of dead-time to satisfy strict specifications can be achieved much better than by integer-order controllers.

To simulate Havriliak-Negami models in the time domain, efficient numerical schemes were developed. Firstly, a convolution quadrature rule was derived on the basis of the Laplace transform representation of the response function. The method allows to discretize fractional Havriliak-Negami models in the time domain and then obtain a numerical approximation useful to simulate the time-domain response of these models. Secondly, a Prabhakar function was employed to describe anomalous relaxation properties of dielectric materials with Havriliak-Negami type of behaviour, doi: 10.1007/s11071-020-05897-9, doi: 10.1515/fca-2020-0002.

Advanced FO control algorithms and realization techniques for automotive applications; model of pain pathways and corresponding software (WG4, D12)

It was shown that some advanced fractional-order control techniques can be applied to automotive engines using the common rail injection system technology and compressed natural gas, which gives a solution to reduce emissions of polluting gases and particulate matter. In this case, the injection process is strongly non-linear, time-variant and highly coupled, so suitable control systems must be designed to guarantee the desired performance. The main controlled variable affecting emissions and consumption of the engines is the common rail pressure in the injection system.

An approach was made available to synthesize and realize fractional order controllers. Synthesis of the controller is based on a loop-shaping technique, which is applied on the open-loop transfer function to achieve frequency-domain performance and robustness specifications. The technique pursues an optimal feedback system in a specified bandwidth and takes advantage of the fractional integrator to achieve enhanced robustness. Moreover, the design approach is reinforced by the *D*-decomposition methodology that guarantees robust stability of the closed-loop system. Finally, the design formulas are specified by closed-form expressions. As regards the realization of the synthesized controllers, accuracy and simplicity are both considered, to allow an efficient and easy implementation as required by industry. Last but not least, the realization formulas guarantee stability and minimum-phase properties of the controllers.

The performance indexes, the robustness (sensitivity to parametric changes) and disturbance rejection capability are tested by simulation of virtual prototypes that are based on very accurate non-linear models of the considered injection systems. Results indicate that fractional-order controllers allow a higher accuracy in metering the injected fuel and better promptness in setting the rail pressure to the desired reference values.

The variation between different working points of the injection system (in terms of the reference rail pressure) is compensated by a model-based fractional-order gain scheduling control strategy, which allows switching from one controller to another each time the working point associated to the rail pressure changes. In case of small variations, only one switch is necessary; if large variations occur, then more controllers are considered such that variations of the injection timings and the rail pressure are limited. In this way, nonlinearity effects, oscillations and instability problems in the rail pressure are prevented.

New results were obtained for the fractional-order control of the common rail pressure affecting the injection process, to increase the performance of an advanced common rail compressed natural gas engine. The reached results contributing to this deliverable were discussed e.g. in doi: 10.1016/j.ifacol.2017.08.2084 and as chapter are part of the Handbook of Fractional Calculus with Applications, Vol. 6, Applications in Control, 2019, Ed. I. Petráš (chapter on Fractional-Order Controllers for Mechatronics and Automotive Applications by P. Lino, G. Maione, pp. 267-292). Additionally, it is expected to publish a manuscript including the latest results in the performance increasement of an advanced common rail compressed natural gas engine in an international peer-reviewed journal relevant to the control engineering community.

Modelling and control of injection systems; Schemes for analyzing propagation of electro-magnetic fields in biological tissues; Report on correlation analysis and corresponding software (WG4, D19)

The contribution to accurate modelling of the electro-injectors that are used in common rail injection systems of Diesel engines was made and presented e.g. in doi: 10.1016/j.ifacol.2016.08.071. The model takes into account the fuel properties, the nonlinear dynamics of the fuel flow, the electro-hydraulic elements and the mechanical components subject to displacement and deformation. But it also considers fractional-order representation of the high-pressure fuel propagation inside a peculiar annular pipe of the electro-injectors, which is given by fractional-order differential equations. To this aim, partial differential equations with fractional-order time derivatives are obtained by starting from the conventional integer-order continuity and momentum equations. It is demonstrated that conservation laws are not violated, thanks to a physical interpretation since the injector is not a closed system hence it loses energy (fractional mass conservation) and on the usage of fractional viscosity (for a fractional momentum balance). The absence of analytical solutions in closed form pushes us to adopt a numerical procedure to solve and simulate the system of time–fractional PDEs by time and space discretization. The specific modelling and control results related to this new approach were presented in a paper submitted to an international peer-reviewed journal. The new model shows a better prediction capability than a rigid body model, which is based on assuming that some relevant coupled mechanical elements behave as rigid bodies, and a nominal model, which uses nominal values of the parameters that are here fixed by conventional expressions but that can be subject to optimization. Model-based simulation shows the improvement in prediction by the help of real data.

Contribution of each working group to the proceedings of the Year 1 Workshop co-located with the Action meeting (all WGs, D5)

The Annual Workshop 2017 was organized in San Sebastian by the Spain representatives of the Action, Dr Karmele Lopez de Ipina and Dr. Pilar Ma Calvo, at University of the Basque Country. Following the tasks and expected deliverables described within the individual Working Groups, 19 speakers from the member countries presented the progress in the description and utilization of fractional-order systems and function blocks. Next to that also representatives of the 3 local companies – Technalia (by Hector Herrero), Stago (by Pablo Martinez Santoja) and OTRI (by Gorka Artola), gave a speech on the possible utilization of fractional-order approach in controlling their designs.

Contribution of each working group to the proceedings of the Year 2 Workshop co-located with the Action meeting (all WGs, D10)

The Annual Workshop 2018 was organized in Bialystok by the Polish representative of the Action, Dr Dorota Mozyrska, at University of Bialystok. Following the tasks and expected deliverables described within the individual Work Groups, 16 speakers from the member countries presented the progress in analysis and design of optimized fractional-order function blocks and system control.

Contribution of each working group to the proceedings of the Year 3 Workshop co-located with the Action meeting (all WGs, D15)

The Annual Workshop 2019 was organized in Ghent by the Belgium representative of the Action, Dr Dana Copot, at Ghent University. Following the tasks and expected deliverables described within the individual Working Groups, 14 speakers from the member countries presented the progress in analysis and design of optimized fractional-order function blocks and system control and discussed their results and future plans. The topics covered the areas from mathematical description to practical utilization in analysis, modelling, classification and/or control of different applications. Newly, the possible usage of fractional approach in cryptography was presented by Fatih Ozkaynak from University of Firat, Turkey. The fractional chaotic systems provide higher entropy and are suitable for more robust cryptography protocols. For more information, please visit Annual Workshop 2019.

Contribution of each working group to the proceedings of the Year 4 Workshop co-located with the Action meeting (all WGs, D20)

The final Annual Workshop originally planned to take place in year 2020 was postponed to March 2021 as based on our request the COST Action period was extended by 6 months due to the COVID-19 pandemic situation. Even if originally planned to be organized as face-2-face in Brno Czech Republic, it was necessary to organize the final Annual Workshop online using the MS Teams platform due to lasting COVID-19 situation. With the end of the COST Action, there were over 50 participants, whereas only 8 active speakers gave their presentations. Even if the number of active speakers was lower competed to previous Annual Workshop, within each presentation there was a fruitful discussion about the recent results and mainly their new research objectives and goals.

Finalization of the book summarizing Action activities and scientific results achieved during its four and half years span (all WGs, D21)

We made it! The last deliverable was reached by finalizing the Action Book that you are just reading. Thank you very much for being interested in activities and hopefully you find inspiration in the results we reached or even in submitting your own proposal for a new COST Action.

Organization of the Action

The management structure of the Action had basically three levels:

(i) Core Group

- Action Chair & Grant Holder Scientific Representative:
  - **Jaroslav Koton** (Brno University of Technology, Czech Republic)
- Action Vice Chair:
  - **Dorota Mozyrska** (Bialystok University of Technology, Poland)
- Science Communication/Dissemination Coordinators:
  - **Harry Esmonde** (Dublin City University, Ireland)
  - **Norbert Herencsar** (Brno University of Technology, Czech Republic)
- STSM Coorinator:
  - **Biljana Jolevska-Tuneska** (Ss Cyril and Methodius University in Skopje, North Macedonia)
- Work Group 1 Leaders:
  - **Igor Podlubny** (Technical University of Kosice, Slovakia); Oct. 2 2016 – Sept. 19, 2019
  - **Sanja Konjik** (University of Novi Sad, Serbia); Sept. 19 2019 – Apr. 2, 2021
- Work Group 2 Leader:
  - **Riccardo Caponetto** (University of Catania, Italy)
- Work Group 3 Leader:
  - **Costas Psychalinos** (University of Patras, Greece)
- Work Group 4 Leader:
  - **Jerzy Baranowski** (AGH University of Science and Technology, Poland)

(ii) Management Committee Members, Substitutes and Observers (see **Action Parties and Representatives**),

(iii) all receiving strong support from **COST Association office**:

- Ralph Stübner: Science Officer,
- Milena Stoyanova: Administrative Officer.

Work Groups

The activities of COST Action and their participants were divided into four Work Groups (WGs), the first dealing primarily mathematical issues as a base for all following Work Groups, the last dealing scenarios of practical utilization of fractional approach:

- WG1 - Fractional calculus and mathematical models:
  improving existing and developing new methods for solving non-integer differential equations; providing application oriented view on the theory of fractional equations to model the behaviour of various function circuits, and control systems; describing the processes investigated within the following WGs.
- WG2 - Fractional-order systems' synthesis and analysis
  focusing on overcoming the lack of reliable solid-state elements featuring fractional-order immitance suitable for hardware realizations of analogue function blocks; presenting tools suited for fractional-order function blocks symbolic and semi-symbolic analysis and description.
- WG3 - Design of analogue and digital fractional-order function blocks
  designing reliable fractional-order function blocks suited for signal processing and control, synthesized both in analogue and digital form.
- WG4 - Utilization of fractional-order systems in engineering and biomedical research areas
  utilizing the fractional order systems in applications rising from engineering and biomedical areas.

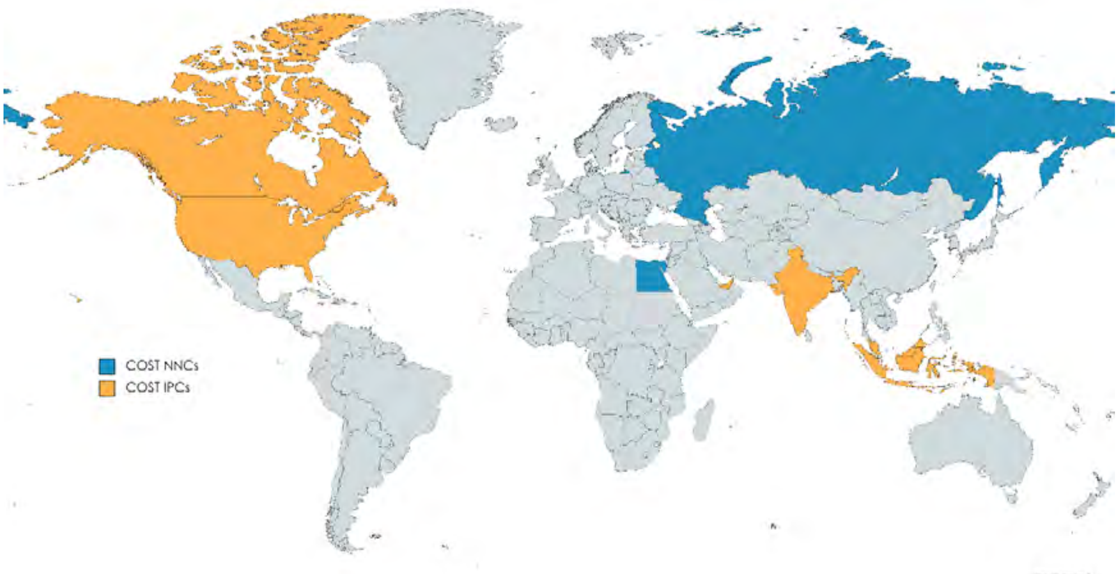
To get a better insight of contribution to each WG, check the Part II of this Action Book, where short papers are present with more links for interested readers.

Action Parties and Representatives

During the period of the COST Action CA15225, it merged people from 25 COST Member Countries – out of 14 were so called Inclusiveness Target Countries (ITCs), and gave the opportunity to interact with research groups from 8 institutions of COST Near Neighbour Countries or COST International Partner Countries.

Each Country was represented by active people accepting the role of being MC Member, MC Substitute or MC Observer.

COST Member Countries



COST NNCs and IPCs

COST Member Countries:

- Belgium
  - Cosmin Copot (Ghent University)
  - Dana Copot (Ghent University)
  - Emmauel Hanert (Université catholique de Louvain)
- Bosnia and Herzegovina (ITC)
  - Fatih Destović (University of Sarajevo)
  - Tarik Uzunovic (University of Sarajevo)
- Bulgaria (ITC)
  - Virgina Kiryakova (Bulgarian Academy of Sceinces)
- Croatia (ITC)
  - Dražen Jurišić (University of Zagreb)
  - Andrej Novak (University of Zagreb)
- Czech Republic (ITC)
  - Norbert Herencsar (Brno University of Technology)
  - Jan Jerabek (Brno University of Technology)
  - David Kubanek (Brno University of Technology)
- Estonia (ITC)
  - Eduard Petlenkov (Tallinn University of Technology)
  - Aleksei Teplakov (Tallinn University of Technology)
- France
  - Catharina Boneet (Inria Saclay Île-De-France)
- Germany
  - Kai Diethelm (University of Applied Sciences)
  - Juergen Kurths (Potsdam Institute for Climate Impact Research)
- Greece
  - Costas Psychalinos (University of Patras)
  - Georgia Tsirimokou (University of Patras)
  - Georce Souliotis (University of Peloponnese)
  - Spyridon Vlassis (University of Patras)
- Ireland
  - Harry Esmonde (Dublin City University)
- Israel
  - Juri Belikov (Israel Institute of Technology)
  - Yossi Keller (Bar Ilan University)
- Italy
  - Riccardo Caponetto (University of Catania)
  - Roberto Garrappa (Università degli Studi di Bari)
  - Guido Maione (Politecnico di Bari)
  - Paolo Lino (Politecnico di Bari)
  - Renato Spigler (Università telematica internazionale Uninettuno)
- Lithuania (ITC)
  - Darius Andriukaitis (Kaunas University of Technology)
  - Grazina Korvel (Vilnius University)
  - Gintautas Tamulevičius (Vilnius University)
  - Algimantas Valinevičius (Kaunas University of Technology)
- Montenegro (ITC)
  - Zana Kovijanic Vukicevic (University of Montenegro)
  - Darko Mitrović (University of Montenegro)

- Netherlands
    - Hassan S. Hoseinnia (Delft University of Technology)
    - Niranjan Saikumar (Delft University of Technology)
- North Macedonia (ITC)
    - Tatjana Atanasova-Pachemska (Goce Delcev University)
    - Biljana Jolevska-Tuneska (University in Skopje)
- Norway
    - Xing Cai (Simula Research Laboratory)
    - Sverre Holm (University of Oslo)
- Poland (ITC)
    - Jerzy Baranowski (AGH University of Science and Technology)
    - Dorota Mozyrska (Bialystok University of Technology)
- Portugal (ITC)
    - Pedro Lima (University of Lisbon)
    - Jose Machado (Instituto Superior de Engenharia do Porto)
    - Maria Luisa Morgado (University of Trás-os-Montes and Alto Douro)
    - Magda Rebelo (Universidade Nova de Lisboa)
- Romania (ITC)
    - Eva-Henrietta Dulf (Technical University of Cluj-Napoca)
    - Eva Kaslik (West University of Timisoara)
    - Cristina Ioana Muresan (Technical University of Cluj-Napoca)
    - Michaela Neamtu (West University of Timisoara)
- Serbia (ITC)
    - Zoran Jelicic (University of Novi Sad)
    - Sanja Konjik (University of Novi Sad)
    - Milan Rapaić (University of Novi Sad)
    - Dušan Zorica (Academy of Arts and Sciences)
- Slovakia (ITC)
    - Ivo Petras (Technical University of Kosice)
    - Igor Podlubny (Technical University of Kosice)
    - Tomas Skovranek (Technical University of Kosice)
    - Jan Terpak (Technical University of Kosice)
- Spain
    - Pilar Mª Calvo (University of the Basque Country)
    - Marcos Faundez-Zanuy (Tecnocampus)
    - Miguel Ángel Ferrer (Universidad de Las Palmas de Gran Canaria)
    - Karmele López de Iniña (University of the Basque Country)
    - Jordi Solé-Casals (University of Catalonia)
    - Blas M. Vinagre (School of Industrial Engineering)
- Turkey (ITC)
    - Baris Baykant Alagoz (Inonu University)
    - Dumitru Baleanu (Cankaya University)
    - Özlem Defterli (Çankaya Üniversitesi)
    - Nusret Tan (Inonu University)
    - Celaleddin Yeroglu (İskenderun Technical University)
- United Kingdom
    - Neville Ford (University of Chester)

COST Near Neighbour Countries

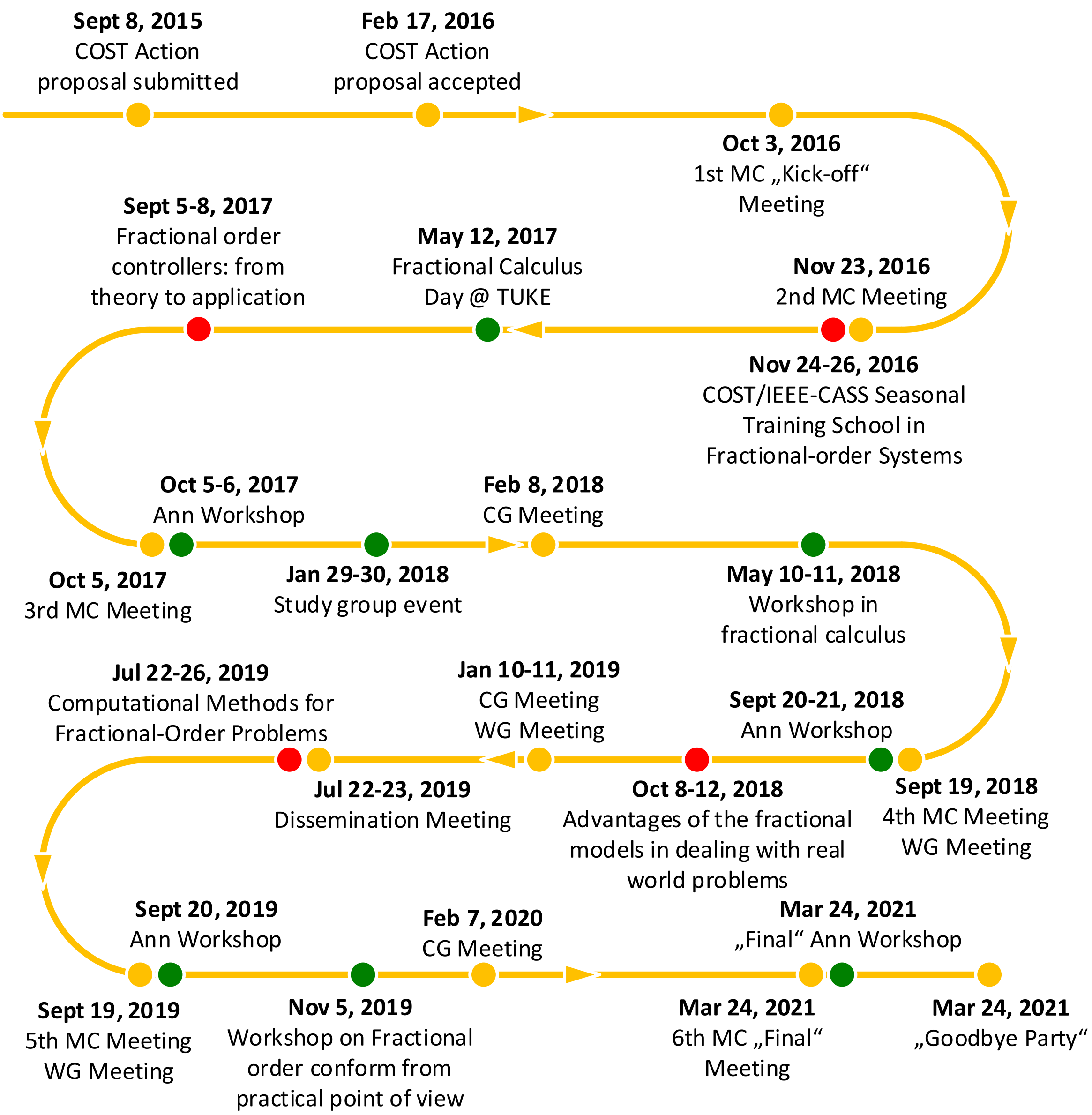
- Egypt, Nile University
    - Ahmed Radwan
- Russian Federation, Kalashnikov Izhevsk State Technical University
    - Peter A. Ushakov

COST International Partner Countries

- Canada, University of Calgary
    - Brent Maundy
- India, Indian Institute of Technology Kharagpur
    - Karabi Biswas
- India, Anand International College of Engineering
    - Praveen Agarwal
- India, University of Kashmir
    - Farooq Ahmad Khanday
- United Arab Emirates, University of Sharjah
    - Ahmed S. Elwakil
- United States, University of Alabama in Tuscalossa
    - Todd J. Freeborn

Organized Events and Activities

A lot did happen once looking at the life-time of COST Action CA15225 during its Grand Periods below. Events were organized and other networking tools supported, such as Meetings, Workshops and Training schools. Somewhere in between also the Short Term Scientific Missions (STSMs) and ITC Conference Grants took place. Check out more by further reading.



Meetings

Meetings were regularly organized to inform, meet and interact with active participants of the COST Action.

The Management Committee (MC) Meetings were held to inform MC Members, Substitutes and Observers about the progress in reaching the Action Deliverables contributing to Action Objectives and to discuss the past and propose future events. In total, six MCs meeting were organized:

- 1st MC “Kick-off” Meeting – October 3, 2016 Brussels (Belgium)

- 2nd MC Meeting – November 23, 2016 Brno (Czech Republic)
- 3rd MC Meeting – October 5, 2017 San Sebastian (Spain)
- 4th MC Meeting – September 19, 2018 Bialystok (Poland)
- 5th MC Meeting – September 19, 2019 Ghent (Belgium)
- 6th MC "Final" Meeting – March 24, 2021 online

We were many already at the beginning, but still made it to grow during the COST Action

Work Group (WG) Meeting were a very efficient tool to discuss inside Work Groups, share and develop joint research ideas, solving the Tasks to contribute and to reach planned Deliverables. The WG Meetings were advantageously organized together with other events, mainly with the MC meetings:

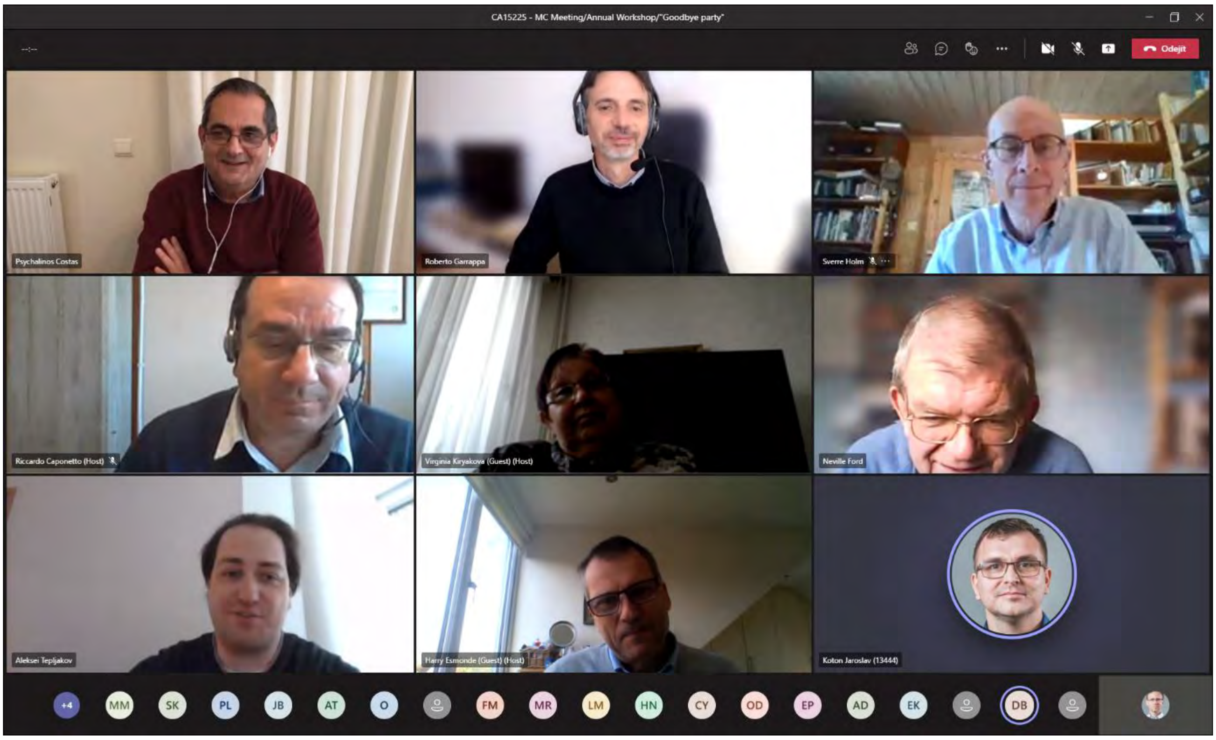
- November 23, 2016 Brno (Czech Republic)
- September 19, 2018 Bialystok (Poland)
- January 10-11, 2019 Dublin (Ireland)
- September 19, 2019 Ghent (Belgium)

Core Group (CG) Meetings were devoted to discuss administrative issues of the Action. The efficiency of the past events organized by the Action were discussed and followed by the pre-approval of the events being planned for the upcoming Grant Period.

- February 8, 2018, Krakow (Poland)
- January 10, 2019 Dublin (Ireland)
- February 7, 2020 Athens (Greece)

In the middle of the COST Action CA15225 life-time, the Dissemination Meeting was organized in a form of invited speech at 2nd International Conference on Electronics and Electrical Engineering July 22-23, 2019 Rome (Italy). Here, the ideas, objectives, activities and reached deliverables of the Action were presented by the Action Chair.

A very special type of Meeting was held at the very end of the COST Action. Unfortunately, due to COVID restrictions still lasting at that time, we had an online "Goodbye Party". As Action Chair I would like to thank to all MC Members, MC Substitutes, MC Observers, our Science Officer and Administrative Officer, and all others, who made it possible to run this COST Action and contributed to its Events and Tasks, used the Networking Tools to follow the Action Objectives and reach its Deliverables.



Screenshot of the "Goodbye Party".

Workshops

To provide the possibility of a more intense interaction and start cooperation even between the Work Groups' members, Annual workshops were organized. Also PhD students were invited to share not just their research breakthroughs and contributions, but also to announce their calls for expertise and help in different fields, not necessarily handled inside their Work Group.

- **Annual Workshop on current progress in fractional-order systems and their utilization**; October 5-6, 2017, San Sebastian (Spain)
  The 1,5-day was organized in San Sebastian by the Spain representatives of the Action, Dr Karmele Lopez de Ipina and Dr. Pilar Ma Calvo, at University of the Basque Country. Following the tasks and expected deliverables described within the individual Working Groups, 19 speakers from the member countries presented the progress in the description and utilization of fractional-order systems and function blocks. Next to that also representatives of the 3 local companies – Technalia (by Hector Herrero), Stago (by Pablo Martinez Santoja) and OTRI (by Gorka Artola), gave a speech on the possible utilization of fractional-order approach in controlling their designs. The book of abstracks can be downloaded here.
- **Annual Workshop on current progress in fractional-order systems, their mathematical description**, modelling and utilization; September 20-21, 2018, Bialystok (Poland)
  The current results and achievements within each Working Group following the Tasks and expected Deliverables of the Action were presented and discussed among researchers and other participants that were present at the two-day event, whereas the total number of participants was 50. The Book of Abstracts can be downloaded here.
- **Annual Workshop**; September 20, 2019, Ghent (Belgium)
  During this Workshop, 14 speakers presented and discusses their results and future plans. The topics covered the areas from mathematical description to practical utilization in analysis, modelling, classification and/or control of different applications. Newly, the possible usage of fractional approach in cryptography was presented by Fatih Ozkaynak from University of Firat, Turkey. The fractional chaotic systems provide higher entropy and are suitable for more robust cryptography protocols. Dr.

Dimiter Prodanov from IMEC research institute, Leuven, Belgium was also invited and contributed with topic on Dirac decomposition for the time-fractional diffusion equation, where he discussed the reduction of the order of the fractional transport equation using Dirac factorization procedure. The Book of Abstracts can be downloaded **here**.

- **"Final" Annual Workshop**, March 24, 2021, online
  The workshop was organized online using MS Teams platform. Although there were only eight active speakers; Roberto Garrappa, Aleksei Tepljakov, Baris Baykant Alagoz, Harry Esmonde, Neville Ford, Dumitru Baleanu, Jaroslav Koton and Todd Freeborn; the discussion that followed each presentation was very active and fruitful. This was mainly due to the fact that not just the latest achievements were presented, but more significantly the future plans of speakers in their research activities. These discussions proved that even if within this COST Action for the last four (four and half) years we tackled the fractional calculus and its possible utilization in various areas, we are still at the beginning.

Next to Annual Workshops, but keeping the intense interaction format were organized.

- **Fractional Calculus Day @ TUKE**, May 12, 2017, Kosice (Slovakia)
  The "Fractional Calculus Day" is One-day workshop in the series of international workshops started in 2007 at the Utah State University, USA. Since then, we had FC Day @ USU (every two years), FC Day @ TUKE (2013, 2014, 2016), FC Day @ IST Lisbon, FC Day @ WUT, FC Day @ UC Merced. This year, the workshop took place at Technical University of Kosice, Slovakia under the official name International Workshop "Fractional Calculus Day @ TUKE".
  This workshop provided lectures by distinguished guests Prof. Jacek Leszczynski (AGH Krakow, Poland), Prof. Richard Magin (University of Illinois in Chicago, USA), Prof. Blas Vinagre (Universidad de Extremadura, Spain), and Dr. Matthew Harker (Montauniversitat Leoben, Austria). Other participants will also contribute their talks. Open problem discussions and round-table discussions took place. The workshop was supported from the funds and projects of Technical University of Kosice without any financial contribution of the COST Action CA15225.
- **Study Group Event**, January 29-30, 2018, Tallin (Estonia)

  This 2-day Study Group event focused on the efficient usage of fractional-order approach in system control domain and signal processing. The event took the advantage of direct interaction and knowledge transfer between academics and industry partners. Based on the problems presented the by the industry to the participants from academia, after brainstorming within the groups the first steps towards possible solutions for the specified problems were discussed together the industry.
  The LDI Innovation and Antasya Software & Consultancy Inc. companies from Estonia and Turkey did participate at the event and provided the issues they were facing. For more information, please visit the web.
- **Workshop on Fractional Calculus – WFC 2018**, May 10-11, 2018, Skopje (North Macedonia)
  Fractional calculus is a modern and expanding domain of mathematical analysis. Using Fractional Calculus in the mathematical models includes more information then offered by the classical integer order calculus. Besides an essential mathematical interest, its overall goal is general improvement of the physical world models for the purpose of computer simulation, analysis, design and control in practical applications. The 2-day event brought together researchers from Bulgaria, Serbia, Macedonia, Algeria, and Montenegro, whereas 18 speeches were given to the audience. The book of extended abstracts can be downloaded **here**.
- **Workshop on Fractional order conform from practical point of view**, November 5, 2019, Delft (Netherlands)
  Frequency domain analysis is a key for industry to understand and design controllers. In this event, the frequency domain analysis tools to design fractional order controllers were presented and discussed within 15 junior and senior researchers. The main contributions were given by: Dr. Duarte Valerio (Universidade de Lisboa), Dr. Niranjan Saikumar (Delft Univeriesty of Technology), Dr. Fabrizio Padula

(Curtin Univeristy), and Dr. Patrick Lanusse (Université de Bordeaux), who within their presentations also gave some practical examples, which advantageously implemented fractional order control efficiently. The presentations of the speakers may be downloaded **here**.

Training Schools

Training Schools (TSs) is another networking took supported by COST Actions. Compared to Meetings, the fact that a TS has the duration of at least 3 days this event makes much broader and deeper interaction possible. In it not about "fast" seeing each other as TS really enables to know each other much better not just during the lectures but also after them having chats e.g. during dinner.

The COST Action CA15225 planned to host five Training Schools. Unfortunately, the last TS with the title "Qualitative theory of fractional-order systems" that should take place in Budva (Montenegro) October 2020 was first postponed to February 2021 but had to be cancelled due to COVID situation at that time. Here we give at least a short list of the remaining four successful Training Schools.

- **COST/IEEE-CASS Seasonal Training School in Fractional-order Systems**, November 24-26, 2016, Brno (Czech Republic)
  This Training School was organized in cooperation with IEEE Circuits and Systems Society and enabled to invite and support Trainers not being MC Observers at that time. The aim of this TS was to bring together Bachelor, Masters, Ph.D. students and as well as experienced scientists working in various interdisciplinary research areas such as advanced mathematics, circuit theory, material science, control, biology, and other up to date topics in fractional-order systems. As Trainers six world-recognized speakers were giving their lectures:
  - **Igor Podlubny** (Slovakia): Foundations of Fractional Calculus for Applications
  - **Ahmed G. Radwan** (Egypt): Approximation and Realization of Fractional-Order Circuits
  - **Riccardo Caponetto** (Italy): Nano Structured Material as Fractional-Order Element
  - **Aleksei Tepljakov** (Estonia): Fractional-order PID Control: Tuning and Practical Implementation
  - **Todd J. Freeborn** (USA): Modeling of Biological Tissues' Properties
  - **Ahmed S. Elwakil** (UAE): Fractional-Order Modeling of Ultra-High Density Capacitors

  Visit the **official website** of this event for more information.

Participants of COST/IEEE-CASS Seasonal Training School (Brno 2016)

- **Fractional order controllers: from theory to application**, September 5-8, 2017, Catania (Italy)
  This Training School was dedicated to the design, implementation and application of FOCs, where the design methods of FOC and of FOC-based control architectures were described and explained in details. Moreover, the realization of FOC was discussed and techniques analyzed to make the trainees aware of

the implementation issues. Finally, within computer labs some real cases were considered to show the application of fractional controllers to different engineering problems of practical interest.
Training School was covered the following trainers and their topics:

- **Igor Podlubny** (Slovakia): Mathematical and computational tools for fractional-order control
- **Blas M. Vinagre** (Spain) : Fractional Order Control: Fundamentals and User Guide
- **Guido Maione** (Italy): Some design approaches of fractional order controllers for electro-mechanical and automotive systems
- **Paolo Lino** (Italy): Modelling and implementation issues of fractional order controllers: Some case studies
- **Vicente Feliu** (Spain): Fractional order robust control of mechatronic systems
- **Duarte Valerio** (Portugal): Fractional Calculus with variable orders
- **Milan Rapaić** (Serbia): Modelling, Identification and Simulation of Non-rational Linear Systems
- **Karabi Biswas** (India): From the definition to the realization of single component fractor
- **Arturo Buscarino** (Italy): Jump resonance fractional order circuits: theory and experiments

The 4-day Training School was attended by 20 Trainees from Spain, Italy, Belgium, Egypt, Czech Republic, Serbia, Montenegro, Poland, and Turkey. For more information, please check the web.

Computer labs on fractional order robust control with Vicente Feliu (Catania 2017)

- **Advantages of the fractional models in dealing with real world problems**, October 8-12, 2018, Istanbul (Turkey)
  The Training School was organized by Dumitru Baleanu and brought together top international specialists from diverse countries and through active participation of the Trainees also initiated fruitful collaboration in the field of fractional dynamics focusing on finding new analytical and numerical methods as well as techniques to model the complexity of the dynamics of some real-world systems. The Training School was providing proves on the advantage of using models based on fractional calculus, contributed to the identification of the unknown phenomena and the stability of the fractional tumor model and may more.
  - **Dumitru Baleanu** (Turkey): Beyond the classical fractional calculus: theory and experimental evidences
  - **Emmanuel Hanert** (Belgium): Application of fractional models to life-science problems: Examples from ecology and pharmacokinetics
  - **Sverre Holm** (Norway): Fractional models in wave propagation and applications in medical imaging and acoustics
  - **S Hassan HoseinNia** (Netherlands): Application of fractional order Control in precision mechatronics
  - **Guido Maione** (Italy): Fractional-order modeling and control in common rail injection systems
  - **Piotr Ostalczyk** (Poland): Vector-matrix description of the variable fractional-order linear systems

- **Milan Rapaic** (Serbia): Real-time identification and parameter estimation in fractional and irrational linear systems
- **Celaleddin Yeroglu** (Turkey): Fractional Order Model Reference Adaptive Control Applications
- **Dorota Mozyrka** (Poland): Discrete-time systems with the Caputo-type fractional order operator – stability issues and applications in consensus modelling.

There were 139 applications to be Trainee at the Training School, where only 20 could be invited and get the support from the COST Action. For more details about this TS, please see the web.

The audience of Advantages of the fractional models in dealing with real world problems (Istanbul 2018)

- **Computational Methods for Fractional-Order Problems**, July 22-26, 2019, Bari (Italy)
A large extent of systems in biology, economics, engineering, physics, and other areas, are modeled by means of differential equations of fractional order whose correct treatment requires the attainment of advanced skills for numerical simulations. The aim of this Training School is to provide to young researchers the background for understanding the mathematics beyond fractional operators and devise accurate and reliable computational methods. In particular, the development of numerical software for the effective treatment of fractional-order systems will be one of the main assets of the training school with the possibility of organizing some laboratory tutorials.
  - **Kai Diethelm** (Germany): Introduction to FDEs, numerical methods for FDEs
  - **Roberto Garrappa** (Italy): Introduction to fractional calculus, efficient implementation of numerical methods for FDEs
  - **Guido Maione** (Italy): Numerical methods in engineering and control theory
  - **Maria Luisa Morgado** (Portugal): Collocation methods for FDEs
  - **Marina Popolizio** (Italy): Matrix methods for FDEs and partial FDEs
  - **Magda Stela Rebelo** (Portugal): Matlab implementation of collocation methods
  - **Abner J. Salgado** (USA): Numerical methods for fractional Laplacian
  - **Martin Stynes** (China): Time-fractional initial-boundary value problems
  - **Yubin Yan** (UK): Numerical methods for fractional partial differential equations

Enthusiastic participants of the Training School (Bari 2019)

Based on the Application forms, the final 40 Trainees from Croatia, Czech Republic, France, Germany, Ireland, Italy, Poland, Portugal, Romania, Serbia, Slovakia, Spain, and Turkey were participating at the Training School, whereas 19 Trainees were financially supported via COST Action CA15225. More information about this TS can be found here.

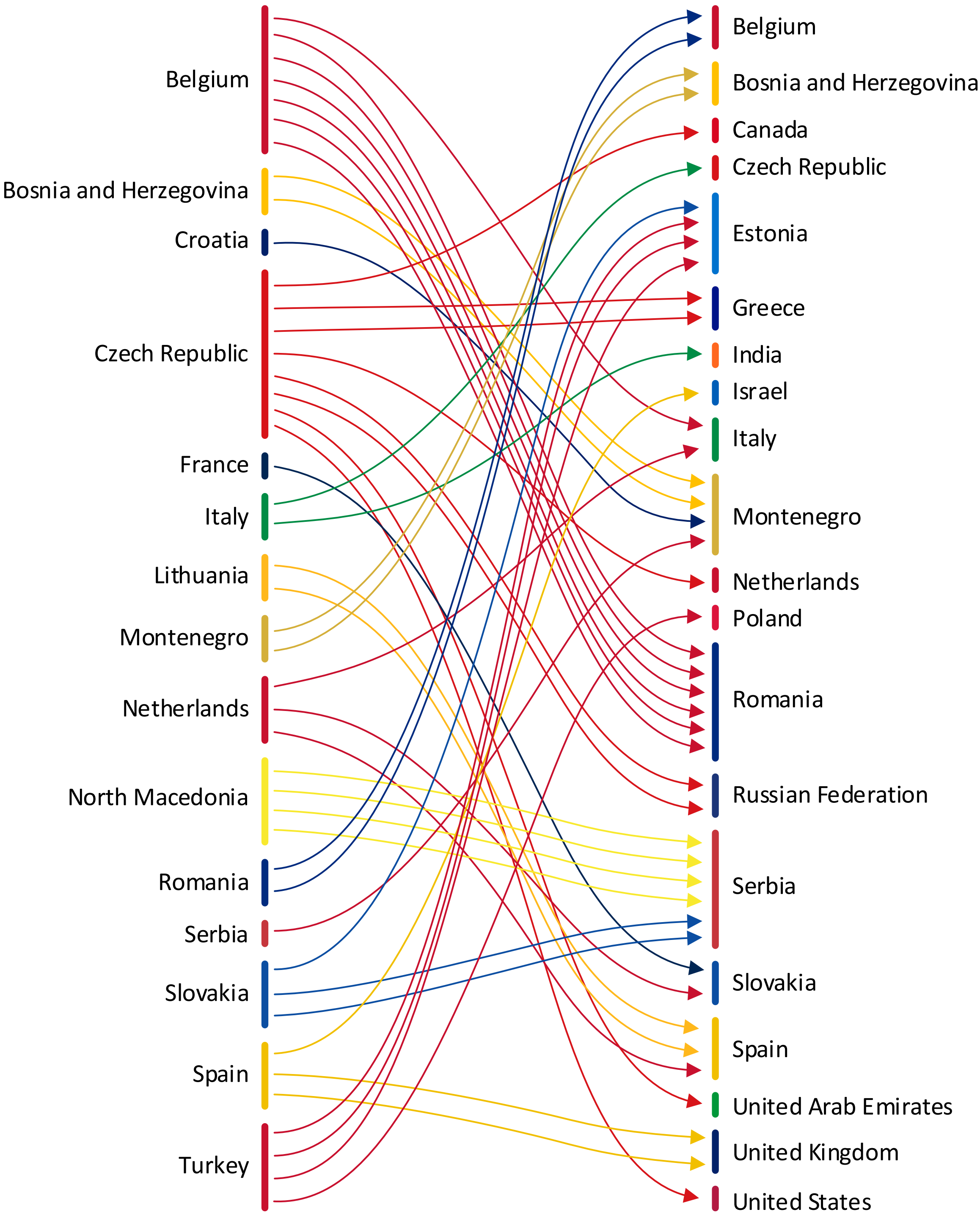
Short Term Scientific Missions

Short Term Scientific Missions (STSMs) are institutional visits aimed at supporting individual mobility, fostering collaboration between individuals. STSM participants were engaged in an official research programme as a PhD Student or postdoctoral fellow or were employed by, or affiliated to, an institution, organization or legal entity which has within its remit a clear association with performing research. The institutions / organization or legal entity, where participants did pursue their main strand of research were considered as Home institutions. The Host institution were the institution / organization that hosted the STSM participant receiving financial support from the COST Action as a Grant.

During the Grant Periods that were not harmed by the COVID, 45 STSMs were granted to support 580 days of participants' missions:

- 7 during 1st Grant Period
- 17 during 2nd Grant Period
- 14 during 3rd Grant Period
- 7 during 4th Grant Period

The most of the STSMs were used to visit institutions and partners within the Europe. Anyway, some also used the possibility to visit colleagues and initiate cooperation with research groups from Canada, India, USA, United Arab Emirates, Russian Federation or Israel. For more details about STSMs supported by the COST Action, please see the full list.



Moving “here and there” thank to STSMs

ITC Conference Grants

The ITC Conference Grants was a new networking tool introduced by COST Association during the 2nd Grant Period of COST Action CA15225. The ITC Conference Grants were aimed at supporting PhD students and ECI researchers from Participating ITC (Inclusiveness Target Country) to attend international science and technology related conferences not specifically organized by the COST Action.

- Piotr Oziablo (Poland), Numerical simulations for fractional variable-order difference eigenfunctions, In proc. 9th Vienna Conference on Mathematical Modelling, Feb. 21-23, 2018, Vienna, Austria, doi: 10.11128/arep.55.a55260
- Oana Brandibur (Romania), Stability analysis of a two-dimensional incommensurate fractional-order conductance based neuronal model, In Proc. Emerging Trends in Applied Mathematics and Mechanics 2018, June 18-22, 2018, Krakow, Poland
- Aleksei Teplakov (Estonia), FOPID Controllers and Their Industrial Applications: A Survey of Recent Results, In Proc. 3rd IFAC Conference on Advanced in PID Control, May 9-11, 2018, Ghent, Belgium, doi: 10.1016/j.ifacol.2018.06.014
- Jan Dvorak (Czech Republic), Design of Fully-Differential Frequency Filter with Fractional-Order Elements, In Proc. 2018 41st Int. Conf. Telecommunications and Signal Processing, July 4-6, 2018, Athens, Greece, doi: 10.1109/TSP.2018.8441259
- Tarik Uzunovic (Bosnia and Herzegovina), Comparison of different methods for digital fractional-order differentiator and integrator design, In Proc. 2018 41st Int. Conf. Telecommunications and Signal Processing, July 4-6, 2018, Athens, Greece, doi: 10.1109/TSP.2018.8441509
- Lukas Langhammer (Czech Republic), Fully-Differential Multifunctional Electronically Configurable Fractional-Order Filter with Electronically Adjustable Parameters, In Proc. 22nd Int. Conf. Electronics 2018, Palanga, Lithuania, doi: 10.5755/j01.eie.24.5.21841
- Andrej Novak (Croatia): Averaged fractional control, In Proc. 12th AIMS Conf. Dynamical Systems, Differential Equations and Applications, July 5-9, 2018, Taipei, Taiwan.
- Norbert Herencsar (Czech Republic), All-Pass Time Delay Circuit Magnitude Response Optimization Using Fractional-Order Capacitor, In Proc. 2018 61st IEEE International Midwest Symposium on Circuits and Systems, August 5-8, 2018, Windsor Canada, doi: 10.1109/MWSCAS.2018.8624059.
- Aslihan Kartci (Czech Republic), CMOS-RC Colpitts Oscillator Design Using Floating Fractional-Order Inductance Simulator, In Proc. 61st IEEE Int. Midwest Symposium on Circuits and Systems, August 5-8, 2018, Windsor, Canada, doi: 10.1109/MWSCAS.2018.8623859.
- Aleksei Teplakov (Estonia), Design of a Generalized Fractional-Order PID Controller Using Operational Amplifiers, In Proc. 25th IEEE Int. Conf. Electronics, Circuits, & Systems, December 9-12, 2018, Bordeaux, France, doi: 10.1109/ICECS.2018.8617954
- Oana Brandibur (Romania), Fractional-order versions of neuronal models, In Proc. 5th Int. Conf. Mathematical NeuroScience, June 23-26, 2019, Copenhagen, Denmark
- Aslihan Kartci (Czech Republic), Synthesis and Design of Floating Inductance Simulators at VHF-Band Using MOS-Only Approach, In Proc. 62nd IEEE International Midwest Symposium on Circuits and Systems, Aug. 4-7, 2019, Dallas, USA, doi: 10.1109/MWSCAS.2019.8885048
- Juan Chen (Estonia), Observer Design for Boundary Coupled Fractional Order Distributed Parameter Systems, In Proc. 7th Int. Conf. Control, Mechatronics and Automation, Nov. 6-8, 2019, Delft, Netherlands, doi: 10.1109/ICCMA46720.2019.8988754

Additional Outputs

Next to the above organized events and supported networking tools, the possibility to better know each other, discuss, share, and broaden the knowledge among the participants enabled the cooperation resulting in join research activities. Not all of them are directly notable, but submitting join project proposals – many of them being later funded, and writing join research papers may be seen as the other visible outputs of the COST Action CA15225.

Join projects

Research projects submitted within international consortia that stemmed from COST Action CA15225 prove the networking within the Action to be functional. Although not all from the below listed projects were later funded, thanks to the Action making the networking possible, the initial step in solving the common issues together is understood as a major impact fully following the African proverb:

If you want to go fast, go alone. If you want to go far, go together.

Projects resulting from Action activities:

- IRES Site: Fractional-Order Circuits and Systems Research Collaboration with EU COST Action, US National Science Foundation, call Standard Grant, proposers: Todd J. Freeborn (US), Jaroslav Koton (CZ); funded, grant No. 1951552, period: 2020-2023 (https://www.nsf.gov/awardsearch/showAward?AWD_ID=1951552)
- Fractional Dynamical Models and Their Applications, National - Scientific and Technological Research Council of Turkey (TUBITAK), proposers: Dumitru Baleanu (TR), Ozlem Defterli (TR); funded, grant No. TBAG 117F473, period 2018-2020
- Microlocal analysis and applications, Serbian Academy of Science and Arts, proposer: Stevan Pilipovic (RS); funded
- Analogue fractional-order systems, their design and analysis, National - Czech Ministry of Education, Youth and Sports, proposer: Jan Jerabek; funded, grant No. LTC18022, period: 2018-2020 https://starfos.tacr.cz/en/project/LTC18022
- Advanced Robust Fractional Order Control of Dynamical Systems: New Methods for Design and Realization – ADFOCMEDER, Translational: bilateral cooperation Italy-Serbia, proposers: Guido Maione (IT), Mihailo P. Lazarević (RS); funded
- Intelligent Control Systems for Industry 4.0, Estonian Research Council, proposer: Eduard Petlenkov (EE); funded, project No. PRG658, period: 2020-2024, https://www.etis.ee/Portal/Projects/Display/79982ae6-2af4-477d-9909-b056b5771dad
- Applied mathematical analysis tools in modeling biophysical phenomena, Bilateral Project Serbia-Croatia, proposers: Sanja Konjik (RS), Davor Horvatić (CR); funded, period: 2019-2020, https://people.dmi.uns.ac.rs/~sanja.konjik/research.html
- Prediction tools for the mechanical behavior of concrete over long time scales, German Federal Ministry of Education and Research, proposers: K. Diethelm (DE), R. I. Leine (DE); funded.
- Fractional-order signal processing and applications, SK-SRB, Slovak Research and Development Agency, proposers: Tomas Skovranek (SK); funded
- Efficient and reliable methods for handling fractional calculus based mathematical models, German Research Foundation, proposers: K. Diethelm (DE), A. D. Freed (US), S. B. Damelin (US); under evaluation
- FRACtional DEVices, HORIZON-ERC-SYG, proposers: Riccardo Caponetto (IT), Biswas Karabi (India), Jaroslav Koton (CZ); not funded
- Design and implementation of fractional-order impedance elements based on distributed resistive-capacitive layered structures, National - Czech Science Foundation, call Standard 2021, proposer: Jan Jerabek (CZ); not funded

- Development of fundamentals of design and implementation of distributed resistive-capacitive elements with fractional impedance, Transnational - Czech Science Foundation, call International 2020, proposers: Jaroslav Koton (CZ), Peter A. Ushakov (RU); not funded
- SCANner for DIAbetes: toward a multi-technology approach for non-invasive glucose monitoring, H2020-FETOPEN-2018-2020, proposers: ES, Andriukaitis Darius (LT), FR, UK, DE, IT; not funded
- Development of fractional-order circuits for biological applications, National – H.F.R.I., proposer: Costas Psychalinos (GR); not funded
- Improving the Modelling of Anomalous Diffusion and Viscoelasticity, National - Portuguese Foundation for Science and Technology, proposers: Luis Ferras (PT), Luisa Morgado (PT); not funded
- Influence of topology, delayed connectivity and memory on the qualitative behavior of dynamical systems on networks and applications, UEFISCDI – Romania, proposers: Eva Kaslik (RO), Mihaela Neamtu (RO); not funded
- Modeling and simulation of the mechanical behavior of CFRP materials, German Federal Ministry of Education and Research, proposers: R. I. Leine (DE), A. Lion (DE), K. Diethelm (DE); not funded
- New mathematical modeling approach of heat and mass transfer phenomena in food engineering in the framework of fractional calculus, Bilateral Project Serbia-India , proposers: Sanja Konjik (RS), Shilpi Jain (India); not funded
- Fundamental research on solid-state fractional-order elements' design, National - Czech Science Foundation, call Standard 2022, proposer: Jaroslav Koton (CZ); not funded

Join publications

Publications were dominant form of join cooperation between very close or even interdisciplinary research groups and individuals being active in COST Action CA15225 activities. Below, there is a list of Journal and conference papers, where some of are used in **Part II** of this Action Book to give an insight to research conducted within this COST Action.

Journal Papers

- Bertsias, P., Psychalinos, C., Radwan, A. G., & Elwakil, A. S. (2017). High-Frequency Capacitorless Fractional-Order CPE and FI Emulator. *Circuits, Systems, and Signal Processing*, 37(7), 2694-2713. doi: 10.1007/s00034-017-0697-0
- Tsirimokou, G., Kartci, A., Koton, J., Herencsar, N., & Psychalinos, C. (2018). Comparative Study of Discrete Component Realizations of Fractional-Order Capacitor and Inductor Active Emulators. *Journal of Circuits, Systems and Computers*, 27(11), 1850170. doi: 10.1142/s0218126618501700
- Abdelaty, A. M., Elwakil, A. S., Radwan, A. G., Psychalinos, C., & Maundy, B. J. (2018). Approximation of the Fractional-Order Laplacian s^alpha As a Weighted Sum of First-Order High-Pass Filters. IEEE Transactions on Circuits and Systems II: Express Briefs, 65(8), 1114-1118. doi: 10.1109/tcsii.2018.2808949
- Kapoulea, S., Psychalinos, C., & Elwakil, A. S. (2018). Minimization of Spread of Time-Constants and Scaling Factors in Fractional-Order Differentiator and Integrator Realizations. Circuits, Systems, and Signal Processing, 37(12), 5647-5663. doi: 10.1007/s00034-018-0840-6
- Koton, J., Kubanek, D., Herencsar, N., Dvorak, J., & Psychalinos, C. (2018). Designing constant phase elements of complement order. *Analog Integrated Circuits and Signal Processing*, 97(1), 107-114. doi: 10.1007/s10470-018-1257-7
- Tepljakov, A., Alagoz, B. B., Gonzalez, E., Petlenkov, E., & Yeroglu, C. (2018). Model Reference Adaptive Control Scheme for Retuning Method-Based Fractional-Order PID Control with Disturbance Rejection Applied to Closed-Loop Control of a Magnetic Levitation System. *Journal of Circuits, Systems and Computers*, 27(11), 1850176. doi: 10.1142/s0218126618501761

- Kubanek, D., Freeborn, T., Koton, J., & Herencsar, N. (2018). Evaluation of (1+ α) Fractional-Order Approximated Butterworth High-Pass and Band-Pass Filter Transfer Functions. *Elektronika Ir Elektrotechnika*, 24(2). doi: 10.5755/j01.eie.24.2.20634
- Kartci, A., Agambayev, A., Herencsar, N., & Salama, K. N. (2018). Series-, Parallel-, and Inter-Connection of Solid-State Arbitrary Fractional-Order Capacitors: Theoretical Study and Experimental Verification. IEEE Access, 6, 10933-10943. doi: 10.1109/access.2018.2809918
- Tepljakov, A., Alagoz, B. B., Yeroglu, C., Gonzalez, E., Hosseinnia, S. H., & Petlenkov, E. (2018). FOPID Controllers and Their Industrial Applications: A Survey of Recent Results. *IFAC-PapersOnLine*, 51(4), 25-30. doi: 10.1016/j.ifacol.2018.06.014
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- Vastarouchas, C., Psychalinos, C., Elwakil, A., & Al-Ali, A. (2019). Novel two-measurements-only Cole-Cole bio-impedance parameters extraction technique. *Measurement*, 131, 394-399. doi:10.1016/j.measurement.2018.09.008
- Despotovic, V., Skovranek, T., & Peric, Z. (2018). One-parameter fractional linear prediction. *Computers & Electrical Engineering*, 69, 158-170. doi: 10.1016/j.compeleceng.2018.05.020
- Machado, J., Kiryakova, V., Mainardi, F., & Momani S. (2018). Fractional Calculus's Adventures in Wonderland (Round Table Held at ICFDA 2018), *Fractional Calculus and Applied Analysis*, 21(5), pp. 1151-1155. doi: 10.1515/fca-2018-0062
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Part II

Mathematical methods of fractional order integration and differentiation

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Introduction

Fractional calculus is a powerful tool for modelling phenomena arising in diverse fields such as mechanics, physics, engineering, economics, finance, medicine, biology, chemistry, etc. It deals with derivatives and integrals of arbitrary real (or even complex) order, thus extending capabilities of the classical calculus, but also introducing novelties in theoretical and applied research. The focus of this task was on enhancement of existing and development of new numerical and analytical methods for derivatives and integrals of fractional orders, and methods for solving ordinary and partial differential equations with derivatives of noninteger orders.

Fractional derivatives and the wave equation

Models that describe waves occurring in the viscoelastic media are given in the form of fractional differential equations supplied with certain initial and/or boundary conditions, and are derived by the rheological analogy from equations that postulate basic physical laws. The presence of fractional derivatives in these models is natural in a sense, due to the viscoelastic character of the media or material under consideration. The latter has been described by different constitutive equations involving derivatives of real and also complex order, such as the fractional generalized Zener, Maxwell or Kelvin-Voigt model. Wave propagation phenomena has been studied on finite and infinite spatial domain. Recent research has revealed that the use of fractional derivatives of complex order in these models could help in resolving some effects that were detected only experimentally and numerically. We were concerned with the questions of solvability and regularity of solutions, thermodynamical restrictions, impact of the initial data and boundary conditions, numerical verifications, etc.

In [1] and [2] we studied generalizations of the wave equation for the case of viscoelastic media described by fractional Zener models using fractional differential operators of complex order. We presented two initial-boundary value problems, investigated them, and provided comparative analysis of techniques used for solving the problems. In both models u, σ and ε denote displacement, stress and strain, respectively, x denotes the spatial coordinate oriented along the axis of the rod and t denotes the time. Further, both models involve various parameters - coefficients and orders of fractional derivatives, whose restrictions follow from the Second Law of Thermodynamics. Viscoelastic properties of the material was modelled by the use of fractional derivatives that were inserted into the constitutive equation. The first problem describes a viscoelastic rod model of finite length l, and consists of a system of equations that corresponds to its isothermal motion, which in the dimensionless form reads:

$$\begin{aligned}
&\frac{\partial}{\partial x}\sigma(x,t)=\frac{\partial^2}{\partial t^2}u(x,t),\\
&\sigma(x,t)+a_1\,{}_0D_t^{\alpha}\sigma(x,t)+b_1\,{}_0\bar{D}_t^{\alpha,\beta}\sigma(x,t)\\
&\quad=\varepsilon(x,t)+a_2\,{}_0D_t^{\alpha}\varepsilon(x,t)+b_2\,{}_0\bar{D}_t^{\alpha,\beta}\varepsilon(x,t),\\
&\varepsilon(x,t)=\frac{\partial}{\partial x}u(x,t),
\end{aligned}\tag{1}$$

$x\in[0,l]$, $t>0$, $0<\alpha<1$, $\beta>0$, together with the initial conditions

$$u(x,0)=0,\ \frac{\partial}{\partial t}u(x,0)=0,\ \sigma(x,0)=0,\ \varepsilon(x,0)=0,$$

and boundary conditions

$$u(0,t) = U(t)\,,\; u(l,t) = 0\,.$$

Initial conditions show that there is no initial displacement, velocity, stress and strain, while boundary conditions prescribe displacement at the rod end points $x = 0$ and $x = l$. The second problem describes a viscoelastic rod model whose length is now infinite. Thus the form of system (1) remains unchanged, with $x \in \mathbb{R}$ instead of $x \in [0,l]$, while the initial and boundary conditions change to:

$$u(x,0) = u_0(x)\,,\; \frac{\partial}{\partial t}u(x,0) = v_0(x)\,,\; \sigma(x,0) = 0\,,\; \varepsilon(x,0) = 0\,,$$

and

$$\lim_{x\to\pm\infty} u(x,t) = 0\,.$$

We proved that the first problem has a unique distributional solution given as

$$u(x,t) = (U *_t K)(x,t) = \int_0^t U(t-\tau)K(x,t)d\tau\,,$$

$x \in (0,l]$, $t > 0$, where

$$K(x,t) = \frac{1}{2\pi i}\int_{s_0-i\infty}^{s_0+i\infty} \exp(ts)\left[\frac{\exp(sM(s)x)}{1-\exp(2sM(s)l)} + \frac{\exp(-sM(s)x)}{1-\exp(-2sM(s)l)}\right]ds\,,$$

with $s_0 > 0$. For the second problem we obtained a unique distributional solution of the form

$$u(x,t) = K(x,t) *_{x,t} \left(u_0(x)\otimes\delta'(t) + v0(x)\otimes\delta(t)\right),$$

$x \in \mathbb{R}$, $t > 0$, where

$$K(x,t) = e^{s_0 t}\left((1-\partial_{xx})G(x,t)\right) + t_+ \otimes \delta(x)\,,\; x,t \in \mathbb{R}$$

and

$$G(x,t) = -\frac{t}{2\pi^2}\int_{-\infty}^{\infty}\int_0^{\infty} \frac{\xi^2\cos(\xi x)}{1+\xi^2}\frac{\tilde{E}(s_0+ip)}{s^2\Psi(\xi,s_0+ip)}e^{ipt}dpd\xi,\; t \geq 0,$$
$$G(x,t) = 0, t < 0,$$

$x \in \mathbb{R}$.

Control problems for fractional differential equations

General problem of control theory amounts to choosing certain control which would lead a system (e.g., governed by a system of (partial) differential equations) from the given initial state to the prescribed final state. Such types of problems have obviously great potential in the sense of various applications and, thus, the control theory is very well developed. It is hard to say where the first mathematical treatment of the problem essentially originates but one can find lots of information in the standard books. However, in certain situation it is not possible to precisely determine the coefficients governing the process and it is natural that the coefficients depend on another (essentially stochastic) variable. In such a situation, we cannot require exact controllability of the system but so-called average controllability. In the contribution [3], we extended mentioned results concerning the averaged control on equations containing fractional derivatives.

We have continued in this direction by considering control problem not only in fractional but also in nonlinear setting. Such kind of questions are of great importance since the most precise way of describing natural phenomena is in the nonlinear framework. The nonlinearity, however, significantly complicates solution procedures since it prevents global apriori estimates. In the frame of the project, through several STSMs and workshops, we were able to introduce a procedure that enables us to connect local estimates and to obtain existence of optimal control. In the essence of the method is local theory of uniqueness of ODEs and Leray-Schauder fixed point theorem. The initial results are presented in [4].

Fractional derivatives in the image processing

Digital image inpainting is the problem of modifying parts of an image such that the resulting changes are not trivially detectable by an ordinary observer. It is used to recover the missing or damaged regions of an image based on the data from the known regions. It represents an illposed problem because the missing or damaged regions can never be recovered correctly with absolute certainty unless the initial image is completely known. In [5] we studied the application of the fractional generalization of the Cahn-Hilliard type equations (CHTE) to the image inpainting problem and proposed a fast algorithm for obtaining its numerical solutions. Through several examples, we showed that fractional PDEs produce superior results over integer order PDEs. Also, we derived a fast algorithm based on the matrix decomposition in the local as well as in the non-local case. In both cases, the idea was to use appropriate arrangements of the discrete equations obtained by the finite difference method so that the computed matrix of the linear system exhibits a sparse structure with block symmetry. This structure enabled us to derive the recursive relations for the computation of the decomposition that, by using simple backward and forward substitutions, yields the solution. We carried out a comparison of this approach with the standard algorithms for numerical solutions of the sparse linear system.

Fractional derivatives and the calculus of variations

The focus was on the optimization of a functional whose Lagrangian depends not only on the integer-order derivatives of the generalized coordinate, but also on its fractional derivatives. Variational formulations of problems are very important in physics. Fractional generalization of the classical theory has found many applications, e.g. in the optimal control. The list of relevant topics within this theory includes well-posedness of constrained and unconstrained fractional variational problems, optimality conditions and the Euler-Lagrange equations, generalizations to higher dimensions, fractional variational symmetries, infinitesimal invariance, Neother's type theorems, fractional conservation laws, discrete-time fractional variational problems, fractional variational calculus on time scales, higher order and isoperimetric problems, complementary fractional variational problems, approximations via the expansion formula, numerical calculations, etc.

In [6] we studied Herglotz variational problems. Herglotz variational principles are of high importance in optimization theory, mechanics and physics, since they provide a consistent method to associate a Lagrangian to the given differential equation such that it becomes the generalized Euler-Lagrange equation of that Lagrangian. We derived optimality conditions for variational problems of Herglotz type whose Lagrangian depends on fractional derivatives of both real and complex order, and resolve the case of subdomain when the lower bounds of variational integral and fractional derivatives differ. Moreover, we considered a problem of the Herglotz type that corresponds to the case when the Lagrangian depends on the fractional derivative of the action and gave an example of the problem that corresponds to the oscillator with a memory. Since our assumptions on the Lagrangian are weaker than in the classical theory, we analyzed generalized Euler-Lagrange equations by the use of weak derivatives and the appropriate technics of distribution theory. Such an example was discussed in details.

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Fractional-order controllers

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Abstract: Fractional-order controllers possess more design flexibility than the integer-order counterparts and may improve the compromise between stability robustness and dynamic performance. This extended abstract synthesizes some relevant results that were obtained, during the period of the COST Action CA15225, in design and application of fractional-order controllers. Some important achievements were possible thanks to the links established with researchers from countries in the COST Action.

Keywords: fractional-order controllers; fractional-order PI/PID controllers; distributed order PID controllers; permanent magnet synchronous motors; fractional-order lead compensators.

Extended abstract

In this abstract, we report a method to optimize controllers for permanent magnet synchronous motors (PMSM). Secondly, we describe the structure and design of a new class of controllers.

PMSM are wide-spread in industry [1] but their control requires fast dynamic response and effective disturbance rejection. The block scheme in Fig. 1 is for the q-axis of the reference frame. PI controllers are usually employed in C_1 and C_2, the first for controlling the stator current component in the reference frame fixed to the rotor (i_{sq}), the second for controlling the rated angular speed of the motor (ω_r).

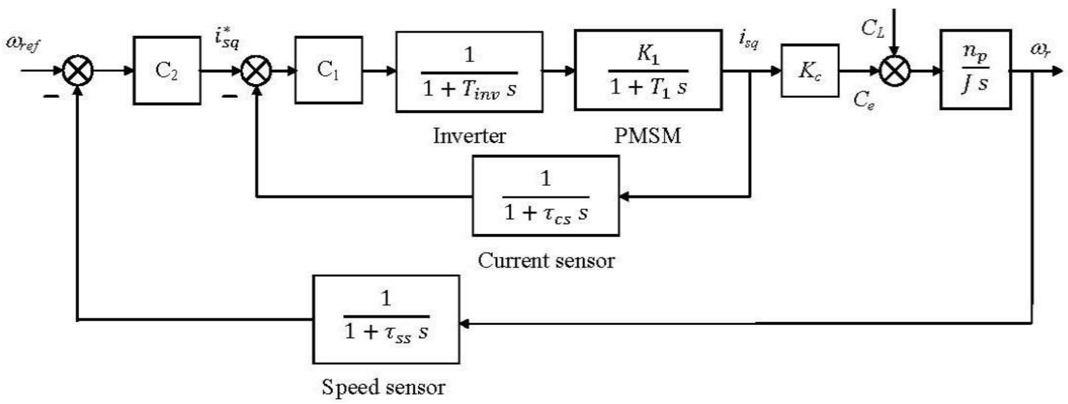


Figure 1: Block scheme of the control system with PMSM

Firstly, we replaced the PI controller in C_2 by a fractional-order PI (FOPI) controller with transfer function $G_C(s) = \frac{K_I(1+T_I s^{\upsilon})}{s^{\upsilon}}$, where υ is the order of integration, whereas C_1 was a PI controller [2]. We designed the FOPI controller by a frequency-domain loop-shaping technique based on robustness and performance specifications. Moreover, we applied a set-point fractional/integer-order pre-filter by a dynamic inversion technique. Subsequently, we considered FOPI controllers for both C_1 and C_2 [3]. The approach allowed to set the parameters of C_1 and C_2 (υ_1, T_{I1}, K_{I1}, υ_2, T_{I2}, K_{I2}) by expressions based on specifications.

In [1], fractional-order proportional-integral-derivative (FOPID) controllers and distributed-order proportional-integral-derivative (DOPID) controllers improved robustness, disturbance rejection, and other performance indexes. The strategy was to optimize the controllers C_1 and C_2 in two successive steps, while using FOPID or DOPID controllers. The FOPID transfer function is

$$G_C(s) = \frac{K_P + \frac{K_I}{s^{\lambda}} + K_D s^{\mu}}{(1+T_f s^{\mu})}, \qquad (1)$$

where λ and μ are non-integer orders, K_P, K_I, and K_D are the controller gains, and T_f is a filtering time constant. If DOPID are considered, then

$$G_C(s) = \frac{\frac{k_0}{s} + \frac{k_1}{s^{2/3}} + \frac{k_2}{s^{1/3}} + k_3 + k_5 s^{1/3} + k_6 s}{1 + T_f s}. \quad (2)$$

To make the control system optimal, we used performance and robustness measures. The maximum noise sensitivity is

$$M_n = \max_{\omega \geq 0} \left| \frac{G_C(j\omega)}{1 + G_C(j\omega) G_P(j\omega)} \right|, \quad (3)$$

where $M_n = K_D / T_f^{\mu}$ for FOPID controllers, whereas $M_n = K_D / T_f$ for DOPID controllers. The maximum sensitivity is

$$M_S = \max_{\omega \geq 0} \left| \frac{1}{1 + G_C(j\omega) G_P(j\omega)} \right|, \quad (4)$$

which is obtained at frequency ω_S. The maximum of the resonant peak is

$$Q = \max_{\omega \geq 0} \left| \frac{K_I G_P(j\omega)/(j\omega)}{1 + G_C(j\omega) G_P(j\omega)} \right|, \quad (5)$$

which is obtained at frequency ω_q.

In the inner loop, to track a current reference, we optimize the controller by maximization of $J_C = \sigma K_I + (1 - \sigma)\omega_B$, with $0.4 < \sigma < 1$, including the integral gain and bandwidth. Then, for FOPID controllers, optimization is defined by

$$\max_{K_P, K_I, K_D, \lambda, \mu, \omega_s, \omega_q} J_C, \quad (6)$$

subject to closed-loop stability and $M_n = K_D / T_f^{\mu} \leq M_{n,\max}$, $M_s \leq M_{s,\max}$, $Q \leq Q_{\max}$. For DOPI controllers, optimization is defined by

$$\max_{k_0, k_1, k_2, k_3, k_4, k_5, k_6, \omega_s, \omega_q} J_C, \quad (7)$$

subject to closed-loop stability and $M_n = K_D / T_f \leq M_{n,\max}$, $M_s \leq M_{s,\max}$, $Q \leq Q_{\max}$.

In the outer loop, speed reference tracking must be accompanied by rejection of step-like disturbances, which is achieved if the controller has as large values of amplitude as possible at all frequencies. Then, we maximize the minimum of the controller amplitude:

$$\max\left\{\max\left\{\left|G_C(j\omega)\right|\right\}\right\}, \quad (8)$$

subject to closed-loop stability and $M_n \leq M_{n,\max}$ $M_s \leq M_{s,\max}$, $Q \leq Q_{\max}$, with $K_P \geq K_{P,\min}$, $K_I \geq K_{I,\min}$ $K_D \geq K_{D,\min}$ for FOPID controllers, and $k_i \geq k_{i,\min}$ $(i = 0,1,2,3,4,5,6)$ for DOPID controllers. We used $M_{n,\max} = 10$, $M_{s,\max} = 2$, and $Q_{\max} = 1.01$.

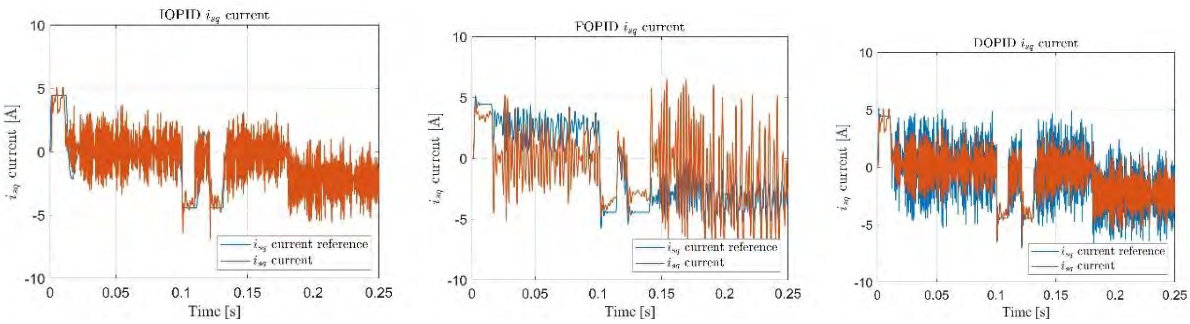
We obtained the first FOPID/DOPID controller (C_1) by considering the plant transfer function $G_1(s) = \frac{K_1}{(1+T_e s)(1+T_1 s)}$, where $K_1 = 1/R_S$ (R_S is the stator resistance), $T_1 = L_S / R_S$ (L_S is the stator inductance), T_e is for all elements in the inner loop. We obtained the second FOPID/DOPID controller (C_2) by considering $G_2(s) = \frac{K_c n_p G_{FOS}(s)}{Js(1+\tau_{ss} s)(1-\tau_{cs} s)}$

for all other elements in the loop, where $G_{FOS}(s)$ is the fractional-order transfer function from the inner closed loop, K_c is the torque constant, n_p is the number of pole pairs, J is the moment of inertia, τ_{ss} and τ_{cs} are the speed and current sensors time constants.

Since optimization is hardly solvable, a Generalized Particle Swarm Optimization algorithm was applied. The optimized FOPID and DOPID controllers were tested by using an accurate nonlinear state-space model of the control system. The model includes uncertainty, noise, parameter variations, and delays [2]-[3].

FOPID and DOPID were compared to PI controllers. The main parameters of the controllers are in Table 1. The phase margins of the inner and outer loops were PM_1 = 32° and PM_2 = 106° with FOPID controllers, PM_1 = 88 and PM_2 = 128° with DOPID controllers. Moreover, FOPID and DOPID controllers reduce the sensitivity. The time response was to a typical speed reference: initially, a step input of 1000 rad/s is applied; secondly, the motion is stopped; finally, the motion is reverted to −1000 rad/s. Moreover, a load disturbance is superimposed during the final phase. The q-axis stator current and the angular speed obtained by the controllers are shown in Fig. 2 and Fig. 3, respectively.

Despite the current oscillations, despite uncertainties and delays, current reference tracking is robustly achieved. In speed control, FOPID and DOPID controllers reduce the maximum overshoot and allow a fast response with zero steady-state error (first phase); PI controllers exhibit a higher undershoot (second phase); DOPID controllers reduce oscillations and response time in the third phase, while rejecting disturbance much better than other controllers. The lowest value of ITAE is 0.52 by DOPID controllers. The control system was also tested against 20% variations of R_S and L_S. Again, the lowest ITAE value (0.55) was given by DOPID controllers, and performance was not much perturbed (see [1]).



(a) (b) (c)

Figure 2: Current in the first test: (a) PI controllers (IOPID), (b) FOPID controllers, (c) DOPID controller

Table 1: Parameters of the controllers

Controller	Parameters						
PI inner loop	K_P = 5.98	K_I = 3.71·10^3					
FOPID inner loop	K_P = 9.36	K_I = 159.20	K_D = 3.41·10^{-4}	λ = 8.86	μ = 1.36		
DOPID inner loop	$k_0 = 8.36$	$k_1 = 0.78$	$k_2 = 0.09$	$k_3 = 1.59$	$k_4 = -0.25$	$k_5 = -0.81$	$k_6 = 0.40$
PI outer loop	$K_P = 0.02$	$K_I = 5.22$					
FOPID outer loop	$K_P = 0.02$	$K_I = 10.81$	$K_D = 2.56 \cdot 10^{-7}$	$\lambda = 0.6$	$\mu = 1.5$		
DOPID outer loop	$k_0 = 2.75$	$k_1 = -0.04$	$k_2 = -0.67$	$k_3 = 6.93$	$k_4 = -0.60$	$k_5 = -0.86$	$k_6 = 0.48$

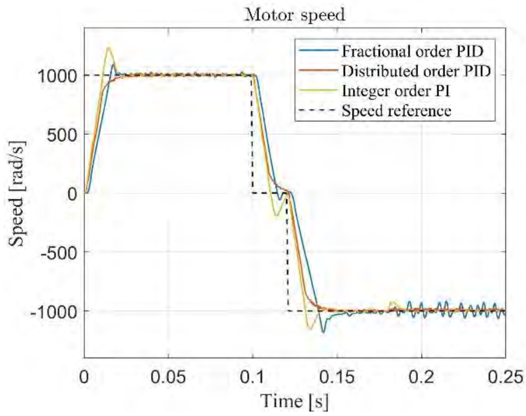


Figure 3: Angular speed in the first test: comparison between PI, FOPID, and DOPID controllers

We also proposed a different controller, named CS-FLEC, in [4]-[7]. It is a fractional-order lead compensator (FLEC) made by the series of two elements that introduce phase leads in shifted and partially overlapping frequency ranges. This connection determines a nearly flat phase in a large frequency range. Moreover, a new method designs a robust controller for a class of benchmark plants that are difficult to compensate because of monotonically increasing lags.

The first element of the CS-FLEC is

$$H_1(s) = \left(\frac{1 + \tau s}{1 + \tau \Delta s}\right)^{\nu_1}, \tag{9}$$

with $0 < \nu_1 < 1$, $\tau > 0$, $0 < \Delta < 1$. The second element is

$$H_2(s) = \left(\frac{1 + \tau \Delta s}{1 + \tau \Delta^2 s}\right)^{\nu_2}, \tag{10}$$

with $0 < \nu_2 < 1$. If $\nu_1 = \nu_2 = \nu$ is chosen, the connection simplifies to

$$H_{12}(s) = H_1(s) H_2(s) = \left(\frac{1 + \tau s}{1 + \tau \Delta^2 s}\right)^{\nu}. \tag{11}$$

The proposed structure is a cascade of two FLECs with the same Bode plots, but the position on the ω-axis of the second stage is shifted with respect to the first one (Fig. 4a). Each element in the CS-FLEC is approximated by a second-order rational transfer function [4], with sufficient accuracy in the frequency range of interest. The fourth-order approximation is advantageous, because the changes of coefficients due to passive component tolerances or to limitations of microprocessor words and quantization effects are contained, such that a low sensitivity to variations is obtained.

The controller $H(s) = K_c\, H_{12}(s)$ is designed in the frequency domain by a practical pattern [4]. The strategy consists in compensating rapidly and monotonically increasing phase lags by the lead introduced in the same frequency range. Firstly, the gain crossover frequency (ω_{pgc}) and phase margin (PM_p) given by the plant are determined and $\Delta = 0.1$ is set.

1. To start, τ is set by $\tau = 1/(\Delta^{0.5}\, \omega_{m1})$, where ω_{m1} is the frequency where $H_1(s)$ gives the maximum lead. However, due to the lead by $H_2(s)$, $H(s)$ reaches the maximum phase lead at $\omega_{m12} > \omega_{m1}$. Hence, to put $\omega_{m12} \approx \omega_{pgc}$, ω_{m1} is chosen such that $\omega_{m1} = h\, \omega_{pgc}$ with $0.5 \le h \le 1$, where h is determined by few attempts, the first being $h = 1$.

2. To set ν_1 and ν_2, the phase margin specification PM is used in $PM = PM_p + \nu_1 55° + \nu_2 16°$. It is suitable to choose $\nu_1 > \nu_2$ to avoid a large shift of the new gain crossover $\overline{\omega}_{gc}$ beyond ω_{pgc}. A rule of thumb is $\nu_1 = 3\,\nu_2$.

3. Then K_c is set such that $K_c \leq 1/\left|H_{12}(j\omega_{pgc})\right|$. The new crossover $\overline{\omega}_{gc}$, indeed, can also be less than ω_{pgc}. This choice is convenient if a greater PM is required and if the slope of the phase diagram in $\overline{\omega}_{gc}$ decreases.

4. The performance is verified. If specifications are not met, then we go back to step 1 (or 3), decrease ω_{m1} (or K_c), change τ, etc.

In [4], we considered two benchmark plant models that are difficult to compensate. Notable performance, low sensitivity and disturbance rejection were obtained. Fig. 4b shows the frequency responses in the case of the second model consisting of an integrator plus a time delay. Note that alternatives in choosing the number of connected FLECs and their free parameters make the CS-FLEC flexible and suitable for difficult control problems.

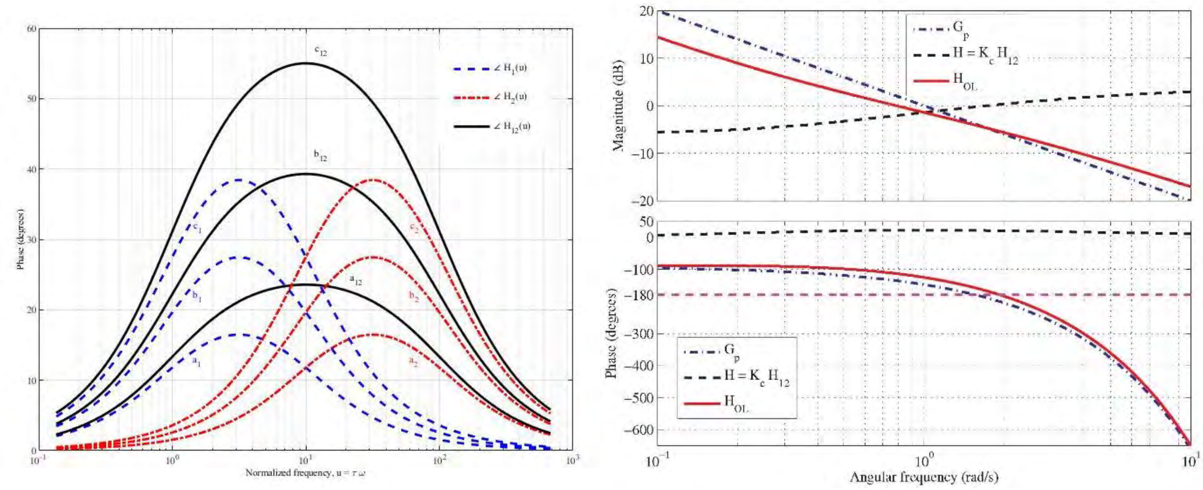


Figure 4: CS-FLEC and compensation: (a) Bode phase diagrams of H_1, H_2, and H_{12} for $\nu = \nu_1 = \nu_2 = 0.3$ (curves a_1, a_2, a_{12}), 0.5 (curves b_1, b_2, b_{12}), and 0.7 (curves c_1, c_2, c_{12}); (b) Frequency response by the integrator plus time delay (dash-dotted lines), controller (dashed lines) and open-loop compensated system (solid lines)

Conclusion and Future Work

Fractional-order controllers design, approximation, and implementation continuously progresses by new strategies and techniques. We proposed some novel ideas regarding design and optimization or new structures that showed to be effective for classes of plants that are hard to compensate. Future work can change or improve the structure of FOPID and DOPID controllers and apply these controllers to different systems. Moreover, it will be possible to analyze new structures of controllers based on fractional-order lead or lag networks and consider special applications involving robotic manipulator and systems.

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Parameter Estimation and Fractional Derivative Modelling of Real Processes

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Introduction

It has been recognised that fractional derivatives are appropriate for modelling materials as they undergo transformation [1]. It has also been shown that fractal structures can be used to represent materials with fractional derivative behaviour [2]. In this work a framework is developed to link both the fractal and fractional approach for modelling transformation processes. An iterative approach is taken to develop a fractal topology that can describe the material structure of phase changing materials. Transfer functions based on fractional calculus are used to describe this topology and then applied to model phase transformations in liquid/solid transitions in physical processes [3]. Two types of transformation are tested experimentally, solidification of gelatine and melting of ethyl vinyl acetate (EVA).

Theory

To begin the analysis an assumption is made on the structure of the material at the start and end of the phase transition. In Figure 1 a material is assumed to have the properties of a Newtonian fluid initially with viscosity η_0 and after transition to have the properties of an elastic solid with stiffness E_1. The complex modulus *G(s)* for the system in transition varies as $G(s)$: $s\,\eta_0 \to E_1$. This transformation can therefore be described by the Transition Function $H_T(s)$

$$H_T(s) = \frac{E}{\eta s} \tag{1}$$

As a process this is represented in Figure 1.

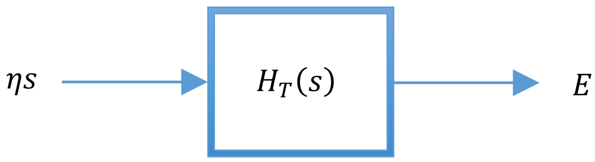


Figure 1 Material transition model

If the change is not instantaneous, then one can use a fractional power β(t) of the Transition Function [4] as shown in Figure 2

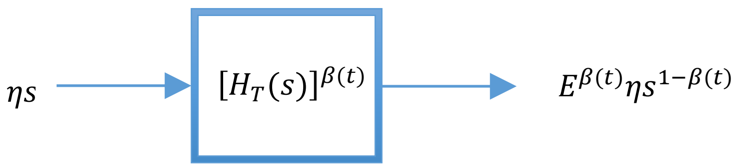


Figure 2 Time dependent material transition model

so that the state of the system $G_\beta(s)$ at time *t* is given by

$$G_\beta(s) = [H_T(s)]^{\beta(t)}\, s\,\eta_0 = \left[\frac{E_1}{s\,\eta_0}\right]^{\beta(t)} s\,\eta_0 = {E_1}^{\beta(t)}(s\,\eta_0)^{1-\beta(t)} \tag{2}$$

This process can be related to fractal systems by first considering the fractal network in Figure 3.

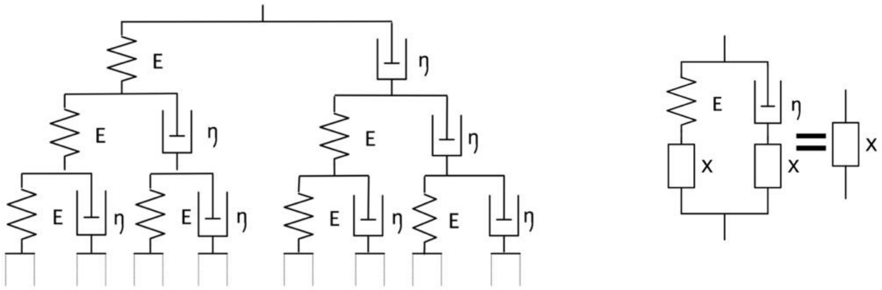


Figure 3 Fractal Network and equivalent system

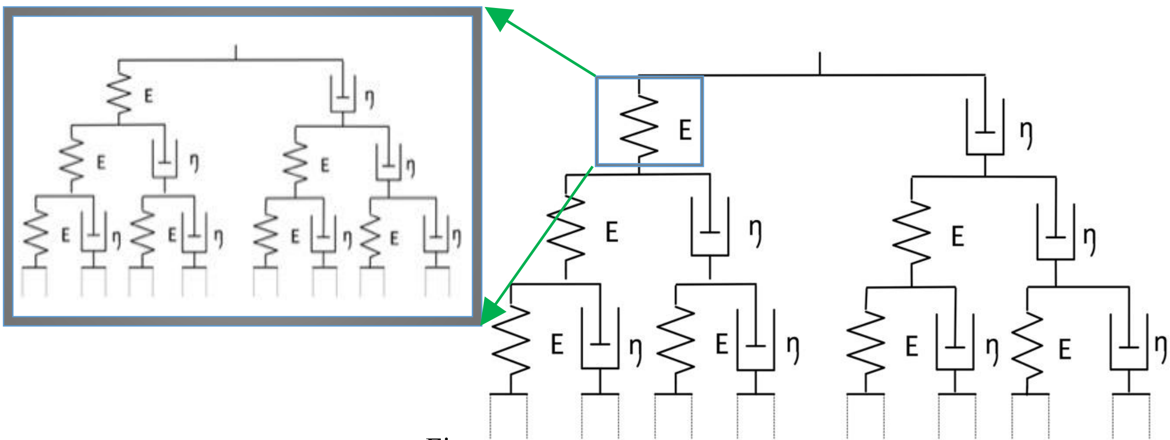
Looking at the structures in Figure 3 one can write

$$X = \frac{1}{\frac{1}{E}+\frac{1}{X}} + \frac{1}{\frac{1}{\eta s}+\frac{1}{X}} \tag{3}$$

so that

$$X = (E\eta s)^{0.5} \tag{4}$$

This gives rise to a fixed fractional order of 0.5. For phase changing materials the fractional power changes as the transformation progresses requiring a more sophisticated model. In Figure 4 a revised fractal network is considered where each spring is replaced by fractal sub network.



Figu

Replacing all the spring elements with the fractal pattern

$$X = \frac{1}{\frac{1}{(E\eta s)^{0.5}}+\frac{1}{X}} + \frac{1}{\frac{1}{\eta s}+\frac{1}{X}} = E^{0.25}(\eta s)^{0.75} \tag{5}$$

This approach can be reiterated with both elastic and viscous components to give a progression of fractional coefficients that follows a sigmoidal pattern as shown in Figure 5.

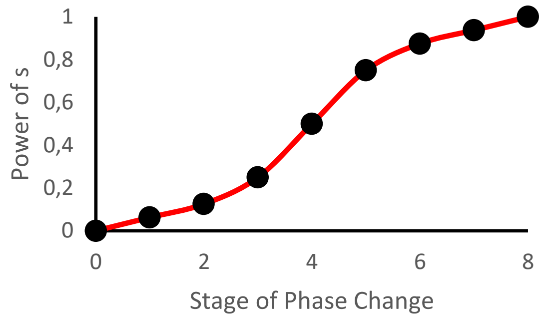


Figure 5 Power of s in fractal evolution

If one iterates the process several times and plots the values the black point values are obtained in Figure 5. In practice it would not be expected that the material phase changes all occur in perfect synchrony. To account for this and still allow for a fractal pattern it is more realistic to consider a random fractal pattern resulting from a distribution of mechanical impedances which shifts as the transformation progresses. The continuous gradual change can then be represented by the red trend line in the Figure 5.

Testing and Results

Testing of the materials was performed using a squeeze film rheometer with 25mm diameter parallel steel plates with a gap of 200mm for gelatine and 1000mm for EVA. The magnitude and phase responses of the gelatine were recorded as cure progressed with time and were recorded for EVA as the material melted as a function of temperature. Fractional models were fitted to the spectra using the generalised reduced gradient solver in Excel

The evolution of the fractional power of β is presented in Figure 6 for gelatine as a function of time and in Figure 7 for EVA as a function of temperature.

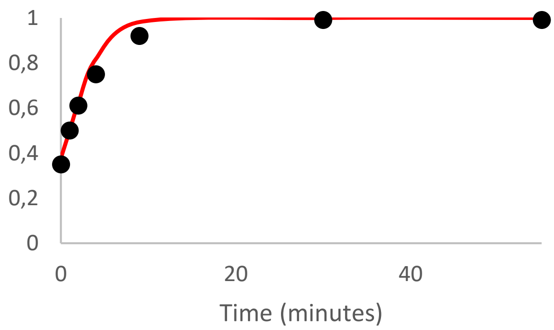


Figure 6 Evolution of $\beta(t)$ for Gelatine

The model for gelatine is based on the transition from a Kelvin Voigt to an elastic material. The evolution of the fractional power shown in Figure 6 does not show the low values at the early stages as seen in Figure 5. This is because when mixing the gelatine, it was necessary to heat the solution to 40°C for one minute prior to testing. Some chemical bonding will have already occurred before testing was undertaken.

The results for EVA were established using two phases. The first was modelled using an elastic to Kelvin Voigt transition and then a Maxwell to viscous transition for the second phase. The evolution of β in Figure 7 does display the full sigmoidal pattern albeit that the transition is over the range 0.1 to 0.4 revealing that a fractional behaviour is also evident at the beginning and end temperatures.

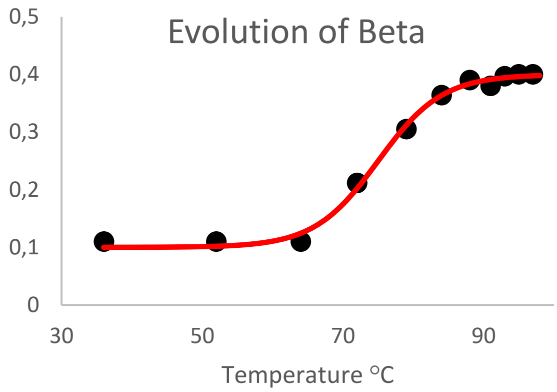


Figure 7 Evolution of $\beta(T)$ for EVA

Conclusion

A fractal topology has been used to describe rheological phase transition in materials. This results in a sigmoidal shaped evolution of material properties that is often seen in practise. The fractal structure can be related to a fractional derivative modelling approach that has been used previously when characterising adhesive cure and which is used here to model the solidification of gelatine and the melting of EVA. The two materials were chosen to represent two different mechanisms of phase transition, chemical bonding for the solidification of gelatine and temperature induced melting for EVA.

Acknowledgement–This article is based upon work from COST Action CA15225, a network supported by COST (European Cooperation in Science and Technology).

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Developing efficient and accurate numerical schemes

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Abstract: Very often solutions of fractional differential equations cannot be represented in a closed form or they have an extremely convoluted representation. Therefore, one of the key issues in fractional-order modeling is the design of efficient numerical schemes for solving differential equations with space and/or time fractional derivatives. In this abstract, we present a brief review of researches and activities related to the Task 1.4. "*Developing efficient and accurate numerical schemes*", in order to highlight the main issues and pitfalls in the development of numerical methods for fractional-order problems and to identify some of the best strategies for an optimal and accurate solution of fractional differential equations.

Keywords: Fractional differential equations, numerical methods, persistent memory term, accuracy.

Extended Abstract

Solutions of fractional differential equations (FDEs) can usually be represented just in terms of very complicated functions (often special functions depending on several parameters) and in several cases, as it is for nonlinear FDEs, there are no analytical solutions which can be represented in a closed form. For this reason, to properly simulate models relying on FDEs it is essential to devise reliable numerical methods and study their main properties.

A Cost Action Training School specifically devoted to "Computational methods for Fractional-order problems" has been organized in Bari (Italy) in 2019. This Training School aimed to provide young researchers with the background for understanding the mathematics beyond fractional operators and devising accurate and reliable computational methods. The presence of trainers active in different areas of numerical methods for fractional-order problems, has allowed to cover the numerical solution of different problems in this fields, such as

- Fractional differential equations
- Space-time partial fractional differential equations
- Fractional Laplacian
- Evaluation of special functions

The Cost Action Training School has also discussed the presence in the scientific literature of several methods which do not appear suitable for solving fractional-order problem and has brought out the need of distinguish reliable from non-reliable methods to be used in fractional calculus. For these reasons some of the trainers of the School have started a joint investigation whose main results have been collected in the paper [1].

Lack of regularity and polynomial approximation

Given an initial value problem for a fractional differential equation

$$ {}^{C}D_{0}^{\alpha} y(t) = f(t, y(t)), \quad y(0) = y_0 \tag{1}$$

it is widely known that the exact solution lacks smoothness since it expands in terms of integer and fractional powers, namely

$$y(t) = \sum_{k=0}^{\infty} Y_k t^{k\alpha} \,,$$

and therefore even the first derivative of the solution $y(t)$ is unbounded at $t = 0$ when $0 < \alpha < 1$.

If not properly considered, this property may lead to unexpected results in numerical simulations. Indeed, most of the numerical methods are based on some kind of polynomial approximation of the solution $y(t)$ or of the vector field $f(t,y(t))$ and polynomials can approximate just in a poor way functions with singularities of this kind.

The lack of regularity of analytical solutions is a general feature of fractional differential equations. In [1] it has been considered a simple test problem such as

$$ {}^{C}D_0^{\alpha} y(t) = f(t), \;\; y(0) = y_0 , $$

and it has been shown that in order to force the solution $y(t)$ to have a certain smoothness, it is necessary to impose that $0 = f(0) = f'(0) = ...$, a condition which appears unnatural and extremely restrictive for the majority of applications. Basically, assuming smoothness of the exact solution in order to derive high-order convergence properties of methods for fractional differential equations appears extremely unnatural.

This is the reason by which it is not reasonable to try to obtain high-order convergence by means of methods based on polynomial approximations.

Main numerical methods for FDEs

In [2] it has been reviewed the main numerical methods available for approximating the solution of FDEs. In particular the following methods have been investigated

- *L1 scheme*: it is obtained after approximating in the FDE (1) with $0 < \alpha < 1$, the first-order derivative in the definition of ${}^{C}D_0^{\alpha} y(t)$ by means of first-order finite difference

$$ {}^{C}D_0^{\alpha} y(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\tau)^{-\alpha} f'(\tau) d\tau, \quad f'(\tau) \approx \frac{f(t_{j+1}) - f(t_j)}{h} $$

  for $\tau \in [t_j, t_{j+1}]$ and $h = t_{j+1} - t_j$.
- *L2 scheme*: it is the generalization of the L1 scheme to FDEs (1) with $1 < \alpha < 2$ and it is obtained by approximating the second-order derivative in the corresponding definition of ${}^{C}D_0^{\alpha} y(t)$ by means of central differences

$$ {}^{C}D_0^{\alpha} y(t) = \frac{1}{\Gamma(2-\alpha)} \int_0^t (t-\tau)^{1-\alpha} f''(\tau) d\tau, \quad f''(\tau) \approx \frac{f(t_{j+1}) - 2f(t_j) + f(t_{j-1})}{h^2} $$

  for $\tau \in [t_j, t_{j+1}]$ and $h = t_{j+1} - t_j$.
- *Product-integration rules*: the are maybe the most used methods for FDEs and the are based on a piece-wise polynomial approximation of the vector field $f(\tau,y(\tau))$ in the integral representation of the solution of the FDE (1)

$$ y(t) = y_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau, y(t)) d\tau $$

  (for simplicity we consider here the case $0 < \alpha < 1$). Constant and linear polynomials are usually considered (as first employed in [2, 3]) since higher-degree polynomials usually do not gives substantial improvement in the accuracy due to the aforementioned lack of regularity of the solution of FDEs.
- *Fractional linear multistep methods*: these methods generalize the standard linear multistep methods for ordinary differential equations and are formulated according to

$$ y_n = y_0 + h^{\alpha} \sum_{j=0}^{\upsilon} w_{n,j} f(t_j, y_j) + h^{\alpha} \sum_{j=0}^{n} \omega_{n-j}^{(\alpha)} f(t_j, y_j) $$

with ω_n the convolution weights and $w_{n,j}$ starting weights introduced to deal with the lack of regularity at the origin. These methods, first introduced in [4]) offer a different way of generalizing to FDEs standard methods for ordinary differential equations but, unlike product-integration, they are able to provide high order of convergence since the introduction of the starting term. The evaluation of coefficients is however more complex than in product-integration rules.

Efficient treatment of the memory term

All the numerical approaches for FDEs have a convolution structure of this kind

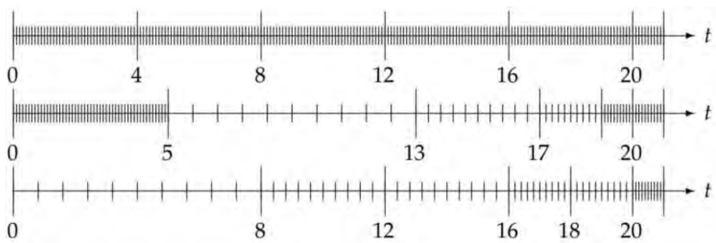
$$y_n = \phi_n + \sum_{j=0}^{n} c_j y_{n-j} \quad \text{or} \quad y_n = \phi_n + \sum_{j=0}^{n} c_j f(t_{n-j}, y_{n-j}),\ n = 1,2,\ldots,N,$$

where the term ϕ_n depends on the initial conditions or other known information. This structure is consequence of the persistent memory of FDEs and of the convolution nature of fractional derivatives and integrals.

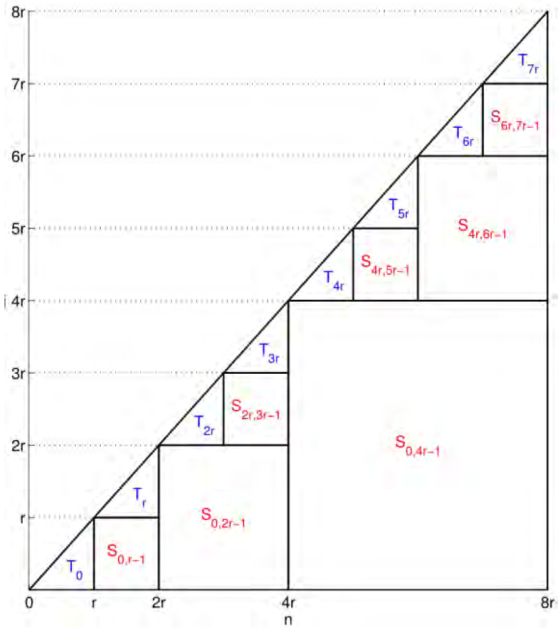
On a mesh-grid of size N, evaluating this convolution in a naive way involves a computational effort proportional to $O(N^2)$ which turns out to be prohibitive in most of the applications.

For this reason efficient algorithms must be considered. The main strategies which have been identified are:

- *Nested mesh techniques*: not all the elements in the above convolution sums are used but some computation effort is saved by including only a subset of them
- *The finite memory principle*: it is based on the finite memory principle by which defined a constant memory length $v > 0$, it is forgotten the entire history of the convolution sum that is more than v units of time in the past; basically the convolution sum is truncated and j runs from $n - v$ to n with the advantage of reducing the computational complexity from $O(N^2)$ to $O(N)$ and the storage needs become constant, although there is a loss of accuracy.
- *Logarithmic memory*: in approaches of this kind distant parts of the memory are not truncated but they are sampled on a coarser mesh and more than one coarsening level is introduced as pictured in the following scheme



- *Fast Fourier Transform Algorithm*: the convolution sum is split in a suitable way thus to directly evaluate just short convolution sums (the triangles in the scheme) and an FFT algorithm is instead applied for the remaining parts of the convolution (the squares in the scheme). This approach does not affect the accuracy of the approximation and reduce the computational complexity form $O(N^2)$ to $O(N(\log_2 N)^2)$. Its application requires a slightly bigger effort in coding [5].



- *Kernel compression schemes*: this term refers to a collection of methods [6, 7] in which the kernel of the fractional integral or derivative is approximated, usually by means of some quadrature rule with exponential weights and the non-local FDE is then replaced by a (possible large) set of local ordinary differential equations, for which standard and very powerful methods are available.

Some of the above mentioned methods can be however applied not only to FDEs (1) but also to slightly different problems.

This is the case of fractional delay differential equations (FDDEs) in the form

$$\begin{cases} {}^{C}D_0^{\alpha} y(t) = f(t, y(t), y(t-\tau)] & , \quad t > 0 \\ y(t) = \phi(t) & , -\tau \le t \le 0 \end{cases} \tag{2}$$

where $\tau > 0$ is a constant delay. In [8] it has been studied the problem of the correct initialization of FDDEs.

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Fractional modelling and optimal control in complex biological systems

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Abstract: Regarding the scientific activities of our team (D. Baleanu (TR-MC Member) and O.Defterli (TR-MC Substitute)), we listed in below a short summary of the published scientific results which contribute to the activities of COST Action CA 15225 such as Mathematical methods of fractional order integration and differentiation, Developing efficient and accurate numerical schemes, Fractionalizing standard models and Approximation of derivatives and integrals of fractional orders by new numerical and analytical methods; Fractionized models.

Extended Abstract

The notion of general classes of operators has recently been proposed [1] (see for example the Prabhakar's class) as an approach to fractional calculus that respects pure and applied viewpoints equally. Several other named models of fractional calculus can fit within the class of operators defined by Prabhakar, and that this class contains both singular and non-singular operators together. It seems that, so far, there is not a unique fractional operator which can be used to describe all types of processes having different types of memory effects [2]. Besides, in the theory of fractional modelling constructed with correct dimensionality it is well-known to follow a so-called five-step method [3].

In the paper [4], we show a fractional-order mathematical model for a tumor-immune surveillance mechanism. We analyze the interactions between various tumor cell populations and immune system via a system of fractional differential equations. An efficient numerical procedure is applied to solve these FDEs by considering various fractional operators. The numerical simulations are presented for different values of fractional order (see Figure 1 for $\alpha = 0.9$) and the asymptotic behavior of the tumor-immune surveillance dynamical system without chemotherapy treatment is presented for all three derivative operators in comparison with a real data set (see Figure 2). Moreover, the Table 1 shows that the absolute and relative errors for the fractional operator with Mittag-Leffler kernel are lower than all other derivative cases. These advantages compensate the additional complexity imposed by the use of the fractional operators in mathematical modeling.

Table 1: Comparative results of the fractional and integer models vs real data

Model	α	Absolute error	Relative error
Caputo	0.952	$1.1271\cdot 10^{-6}$	0.0265
CF	0.92	$1.3246\cdot 10^{-6}$	0.0311
ABC	0.9	$2.9030\cdot 10^{-5}$	0.0068
Integer	1.0	$4.9501\cdot 10^{-6}$	0.1164

Additionally, an optimal control strategy is imposed into the model to examine the effect of chemotherapy treatment. Simulation results show that the new presented model represents various asymptomatic behaviors that tracks the real data more accurately than the other fractional- and integer-order models. Numerical simulations also verify the efficiency of the proposed optimal control strategy and show that the growth of the naive tumor cell population is successfully declined.

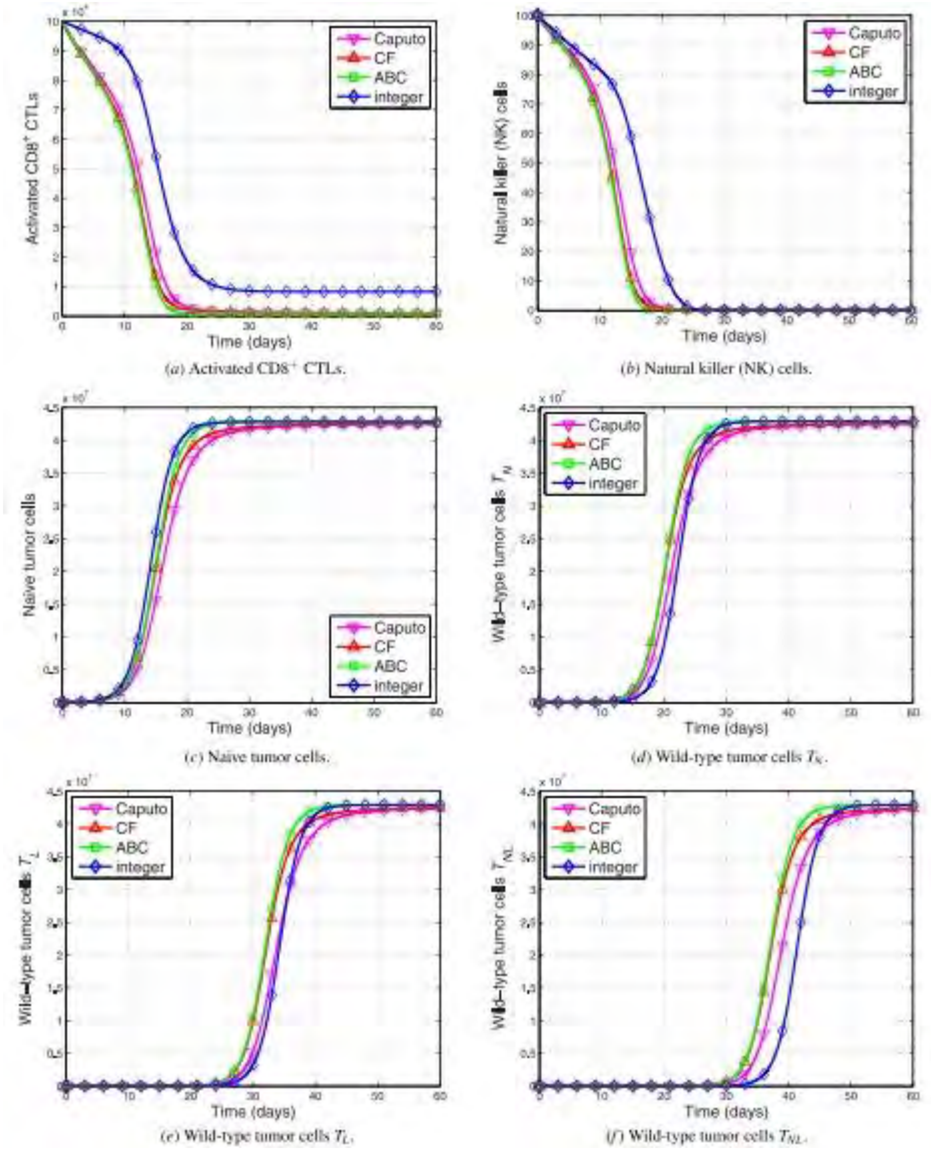

Fig 1: The growth of immune and tumor cell population derived by the three different fractional models with α = 0.9. [4]

In the papers [6, 7], we study a deterministic mathematical model from epidemiology which is about anticipating the influence of temperature on dengue transmission incorporating temperature-dependent model parameters. This model is investigated within different definitions of fractional operators such as Caputo and Mittag-Leffler and numerical simulations are obtained by two different numerical schemes for various fractional orders comparatively.

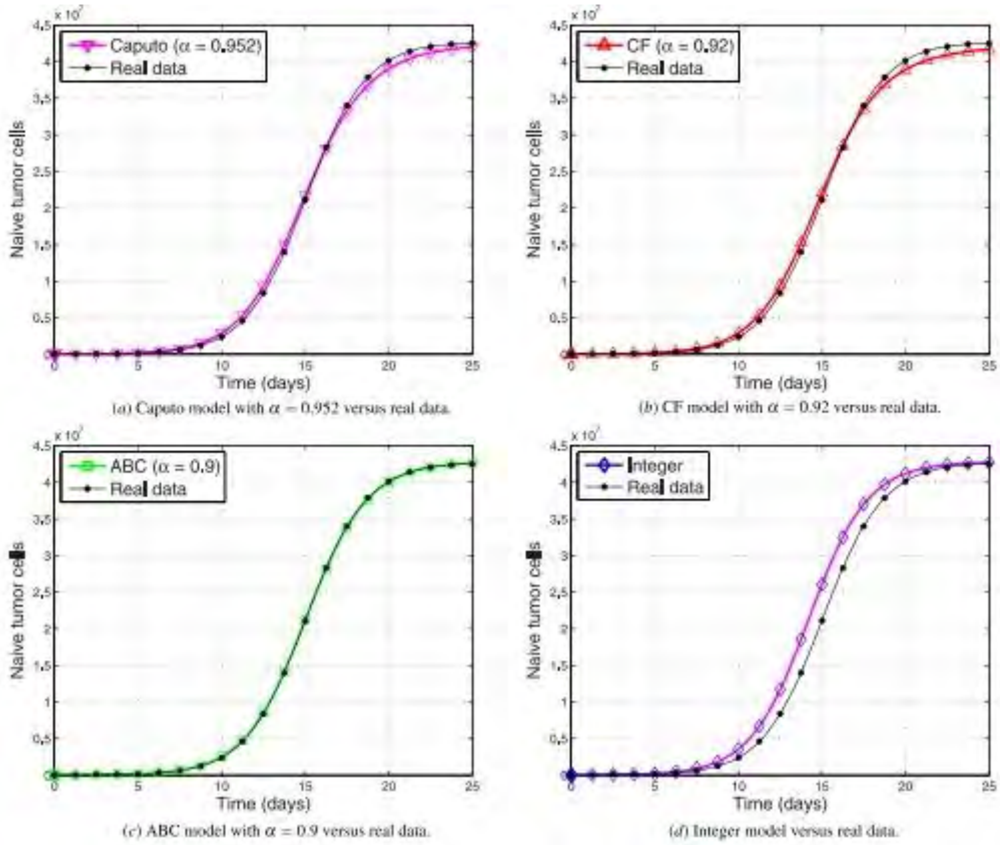

Fig. 2: The growth of the naive tumor cell population derived by three different fractional models and integer model vs the real data of tumor growth used by [5]

Regarding the scientific activities of our team (D. Baleanu (TR-MC Member) and O.Defterli (TR-MC Substitute)), we listed in below the related national scientific projects, organized scientific events and STSM that are performed in the name of the COST Action CA 15225:

Related National Projects

- Title "Fractional Dynamical Models and Their Applications"- Grant No: TUBITAK TBAG 117F473 (Duration: April 2018-October 2020) Proposers: D. Baleanu(TR), O. Defterli(TR).Granted by Scientific and Technological Research Council of Turkey(TUBITAK) - with budget 117525TL.

Organized International Events

- *COST CA15225 – Training School – Advantages of the fractional models in dealing with real world problems* Link: (https://fractional-systems.eu/ts-2018/)

  The five-day Training school that took place in the Istanbul Gelisim University, Istanbul, Turkey and was organized by Dumitru Baleanu brought together top international specialists from diverse countries (members of our COST Action) and through active participation of the Trainees also initiated fruitful collaboration in the field of fractional dynamics focusing on finding new analytical and numerical methods as well as techniques to model the complexity of the dynamics of some real-world systems. There were 139 applications to be a Trainee and 20 Trainers at the Training School. The Training School was providing proves on the advantage of using models based on fractional calculus, contributed to the identification of the unknown phenomena and the stability of the fractional tumor models. New software for solving fractional differential equations and fractional discrete equations was provided. Furthemore, the discussion on optimal control theory took place providing a powerful tool to link biological, mechanical or physical requirements coming from the system under investigation to the required mathematical objectives. With the help of a very recently established fractional derivative the working plan for example control problems were designed.

- *International Conference on Computational Methods in Applied Sciences*, IC2MAS2019, Istanbul, Turkey; Organizing a special session entitled ”Fractional BIO-MATH” with Chair Ozlem Defterli and dedicated to COST CA 1525. Link: (https://ntmsci.com/Conferences/ICCMAS19/Announcements).

  In this special session, new studies on the analysis and modeling of biological processes having complex dynamics were discussed with the help of classical and newly proposed fractional operators. There were 4 main plenary lecturers and 25 participants. The applications in different sub-branches of biology were discussed within interdisciplinary relations. In this respect, the performance of fractional derivative and integral operators with classical derivative and integral operators were compared through some important new applications on real world biological problems. New developments in the related field were discussed.

STSM Activities

- COST CA 15225 – STSM (Short Term Scientific Meeting) between Cankaya University-Turkey and (Host) Bialystok Technical University-Poland, 12 - 17 March 2018.

  Title: Modelling cancer tumors by fractional calculus Duration: 12 - 17 March 2018 Visitor: Dr. Dumitru Baleanu (MC Member-TR) Host: Dr. Dorota Mozyrska (MC Member-PL)

  The purpose of the STSM was to bring together participating top international COST members specialists from Turkey (Dumitru Baleanu, Cankaya University) and Poland (Dorota Mozyrska and her team, Bialystok University of Technology, Poland) and to developed fruitful collaborations in the field of fractional dynamics focusing on finding new analytical and numerical methods as well as techniques to model the complexity of the dynamics of cancer tumours. On March 15th, 2018, Dumitru Baleanu has presented a general seminar entitled “Fractional calculus with applications: 50 years of Caputo derivative and the Faculty members were the audience. As a result of this STSM a common published paper (DOI:10.1063/1.5096159) was reported.

- COST CA 15225 – STSM (Short Term Scientific Meeting) between Cankaya University-Turkey and (Host) Ghent University-Belgium.

  Title: “New Perspectives in Computational Biology with Applications in Cancer Research” Duration: 31.01.2020-07.02. 2020

  Visitor: Dr. Ozlem Defterli (MC Substitute-TR) Host: Dr. Dana Copot (MC Member-BE)-Dynamical Systems and Control (DYSC) Research Group in Department of EMMCS, University of Ghent, Ghent-Belgium.

  Dr. Dana Copot’s (Belgium-MC Substitute) research team with two Master and Ph.D. students and Department members were the audience of the two lectures given by Dr. Ozlem Defterli (Turkey-MC Substitute). Dr. Ozlem Defterli presented the following seminars/lectures during the STSM activity at Ghent University:

  Seminar I: ”New Perspectives in Computational Biology with Applications in Cancer Research-PART I” Date&Time: 05.02.2020, Wednesday, 10:00 am

  Seminar II: ”New Perspectives in Computational Biology with Applications in Cancer Research-PART II” Date&Time: 07.02.2020, Friday, 10:00 am Place: UGent-EMMCS-DYSC, Campus Ardoyen, Technologiepark 125, 2nd floor, 9052 Zwijnaarde.

Interaction and exchange of research ideas was made on the topic of cancer research, tumor-growth within two research groups, organization of common scientific events is planned in the near future.

Acknowledgement– This work is based upon work from COST Action CA15225, a network supported by COST (European Cooperation in Science and Technology).

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Development of MATLAB toolboxes for fractional-order problems

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Abstract: Matlab is one of the leading environments for performing scientific calculations, used by scientists in many fields, especially in engineering for the control system design. For this reason it appears natural that most of the numerical methods studied and devised in the frame of this Cost Action are implemented in Matlab toolboxes. The creation of Matlab toolboxes, an activity of the Task 1.5 "*Development of toolboxes for MATLAB*" is therefore the obvious consequence of the Task 1.4. Matlab, as well as any other program language or scientific package, does not provide any facility for solving fractional-order problems, thus the outcomes of this Task are expected to be of great help for the scientific community involved in the simulation of fractional-order model. All the codes realized in this task are made freely available to the whole community since they are uploaded on the website MATLAB Central File Exchange maintained by MathWorks, Inc., the makers of MATLAB, thus ensuring large accessibility.

Keywords: Fractional differential equations, numerical methods, matlab codes.

Extended Abstract

The development of accurate and reliable numerical methods for solving fractional differential equations and related problems is not always sufficient to allow non specialists to simulate, in a reliable and efficient way, systems described by means of fractional-order operators.

The implementation of numerical methods for fractional-order problems, even when described with great details in the scientific literature, is indeed not always simple and must take into account a number of factors such as the dependence on the fractional order, the need of treating in reasonably fast way the memory term and others.

Unfortunately, unlike ordinary differential equations for which several built-in functions are available, the main scientific packages and the main programming languages do not provide any kind of support for fractional-order problems. In most cases, researchers with a not specific expertise in programming numerical codes have to be involved, with great difficulties, in writing by themselves codes for checking results and correctness of their models.

For these reasons, one of the main activities of the Cost Action, planned in the Task 1.5 "*Development of toolboxes for MATLAB*", has been the creation of a set of specific Matlab codes for solving a series of fractional-order problems with the aim of making them available to the whole scientific community.

The codes have been uploaded in the official Mathworks repository which can be easily accessed from any user; their download and use is free for anyone.

The choice of Matlab as programming language is related to the fact that this language is one of the leading environments for performing scientific calculations and it is also widely used for control system design, one of the main topic of this Cost Action. The translation in other programming languages, or codes for other platforms, is however possible and has been already done (in some limited cases) from researchers outside the Cost Action.

The main features of all codes specifically developed for fractional-order problems in the framework of this Cost Action task are the following

- *generality*: they solve not just a single or specific equation but an as wider as possible class of problems;
- *easy-to-use*: the interface of these codes is in most cases the same interface of similar codes for integer-order problems that Matlab users are used to;
- *reliability*: codes provide accurate results and it is allowed to users with more expertise to take under control the desired accuracy by properly setting specific parameters.

All these Matlab codes can be freely downloaded from

https://www.mathworks.com/matlabcentral/profile/authors/2361481

Here follows a short description of each code and of the problems which they solve.

Predictor-corrector PECE method for fractional differential equation

This code solves initial value problems for fractional differential equations according to the method first introduced in [1, 2] and based on a couple of explicit and implicit Adams-Bashforth-Moulton methods obtained from product integration rules (see also [3]).

In order to allow a fast execution, and avoid the long time execution required by the persistent memory, the FFT algorithm described in [4] is implemented.

The use of this code follows a syntax similar to other built-in Matlab codes for solving integer-order differential equations.

```
[T,Y] = FDE12(ALPHA,FDEFUN ,T0,TFINAL ,Y0,H)

% ALPHA    : fractional order
% FDEFUN   : function handle for the FDE vector field
% T0       : starting point
% TFINAL   : final point of integration
% Y0       : initial value
% H        : step-size for numerical integration
```

Product integration for multi-term fractional differential equations (MFDEs)

This code solves multi-term fractional differential equations (MFDEs) with a linear or a nonlinear term

$$\lambda_Q D^{\alpha_Q} y(t) + \lambda_{Q-1} D^{\alpha_{Q-1}} y(t) + ... \lambda_1 D^{\alpha_1} y(t) = f(t, y(t))$$

by means of a product-integration rule of implicit type. A description of the code is available in [5]. Also in this case the efficient treatment of the memory term is assured by the FFT algorithm.

The syntax for the call of this Matlab code follows the particular nature of the problem at hand

```
[t, y] = MT_FDE_PI1_Im(ALPHA,LAMBDA ,F_FUN ,J_FUN,T0,TFINAL ,Y0,H)

% ALPHA    : vector of fractional orders
% LAMBDA   : coefficients of the multi-term equation
% F_FUN    : function handle for the nonlinear term
% J_FUN    : function handle for Jacobian of the nonlinear term
% T0       : starting point
% TFINAL   : final point of integration
% Y0       : initial value
% H        : step-size for numerical integration
```

Solution of fractional delay differential equations (FDDEs)

The code solves fractional delay differential equations (FDDEs) with one constant delay of linear and nonlinear type

$$^{C}D_0^{\alpha} y(t) = g(t, y(t), y(t-\tau)), \qquad y(t) = \phi(t) \;\; t \in [t_0 - \tau, t_0]$$

by means of the first-order product-integration rule described in [6], where the effects of initial conditions have been studied.

The description of the main parameters of the code is the following.

```
[T, Y] = FDDE_PI1_Ex(ALPHA,F_FUN ,TAU,T0,TFINAL ,PHI,H)

% ALPHA   : fractional order
% F_FUN   : function handle for the nonlinear term
% TAU     : constant time delay
% T0      : starting point
% TFINAL  : final point of integration
% PHI     : initial value function
% H       : step-size for numerical integration
```

Evaluation of the Mittag-Leffler function

This code evaluates the Mittag-Leffler function

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}$$

for any (possible complex) argument z and for any $\alpha > 0$ and β. It is used an algorithm previously discussed in [7] and based on quadrature rules applied to the integral formulation obtained from the inversion of the Laplace transform

$$E_{\alpha,\beta}(z) = \int_C e^s \frac{s^{\alpha-\beta}}{s^\alpha - z} ds$$

where C is a parabolic contour in the complex plane suitably chosen to obtain high accuracy in a fast way.

The Mittag-Leffler function plays an important role in the solution of fractional differential equations and in the analysis of their stability properties [8]. Although, its importance in fractional calculus, non specific codes for the evaluation of this function were available in the main programming languages.

The function is able to compute the Mittag-Leffler function with 3 parameters (also known as the Prabhakar function)

$$E_{\alpha,\beta}^{\gamma}(z) = \frac{1}{\Gamma(\gamma)} \sum_{k=0}^{\infty} \frac{\Gamma(\gamma + k) z^k}{k! \Gamma(\alpha k + \beta)}$$

but just for a restricted range of arguments and parameters.

The use of this code is dictated by its parameters and follows the following syntax.

```
function E = ml(Z,ALPHA ,BETA)

% Z     : argument of the Mittag -Leffler function
% ALPHA : first parameter
% BETA  : second parameter
```

The argument z must be a scalar (real or complex) or a vector (in the last case the code returns a vector with the values of the Mittag-Leffler function in each entry of the vector). It has been however released a further code for the evaluation of the Mittag-Leffler function with matrix arguments

$$E_{\alpha,\beta}(A) = \frac{1}{\Gamma(\gamma)} \sum_{k=0}^{\infty} \frac{1}{\Gamma(\alpha k + \beta)} A^k \qquad A \in C^{n \times n}$$

whose use is similar to the code for the scalar case (except for the fact that the argument can be any square matrix).

```
function E = ml_matrix(A,ALPHA ,BETA)

% A     : matrix argument of the Mittag -Leffler function
% ALPHA : first parameter
% BETA  : second parameter
```

By means of the the matrix Mittag-Leffler function it is possible to solve directly linear systems of fractional differential equations or develop more efficient methods for general systems of fractional differential equations.

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An Approach for Modelling Time-Varying Systems Using Fractional Derivatives

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Introduction

The dynamic behaviour of systems can be represented in terms of magnitude and phase values using frequency response functions. Systems that display a constant phase that is not an integer multiple of p/2 over large bandwidths imply fractional derivative behaviour [1]. In this abstract a new technique is used to formulate fractional models of time-varying systems based on an understanding of the pre and post transformation states. As an example, the dynamic response of adhesive during cure is examined using squeeze film rheometry [2].

Theory

To begin the analysis an assumption is made on the structure of the material at the start and end of the phase transition. In Figure 1 a material is assumed to have the properties of a Newtonian fluid initially with viscosity η_0 and after transition to have the properties of an elastic solid with stiffness E_1. The complex modulus *G(s)* for the system in transition varies as $G(s)\colon\ s\,\eta_0 \rightarrow E_1$.

For most material phase changes the transition takes a finite amount of time and therefore there will be interstitial stages during the process. One way of determining the state of the material during transition is to formulate a transition function $H_T(s)$ and then multiply the initial modulus by fractional powers of this function. The transition function is obtained by formulating the ratio of the final state to the initial state. In the case of the system in Figure 1 this is

$$H_T(s) = \frac{E_1}{s\,\eta_0} \tag{1}$$

If the transition has progressed by a fraction β ($0 \le \beta \le 1$), then the state of the material is given by the complex modulus $G_\beta(s)$

$$G_\beta(s) = [H_T(s)]^\beta\, s\,\eta_0 = \left[\frac{E_1}{s\,\eta_0}\right]^\beta s\,\eta_0 = {E_1}^\beta (s\,\eta_0)^{1-\beta} \tag{2}$$

The value of β will vary as time progresses starting at 0 and eventually reaching 1 when full transition has occurred. Between these values the fractional value will give rise to fractional derivative behaviour since the Laplace operator is raised to a fractional power [3].

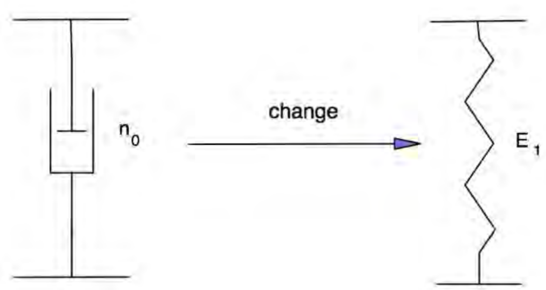

Figure 1 Material transition model

For this technique to be relevant there has to be a phase change in the system, in other words a change in the ratio of real to imaginary components in the complex modulus. This will obviously be the case for a curing adhesive, but the technique could be applied to any system described by a changing complex parameter.

To use the procedure outlined above, an appropriate choice of start and finish structure is required. This is done by examining the dynamic modulus of the material at the start of transition and at the end. For a methacrylate adhesive a Maxwell system is chosen for the initial configuration while a standard linear solid (SLS) model is selected for the end state. Thus, the adhesive curing process can be regarded as a transition from a Maxwell to a SLS system, see Figure 2

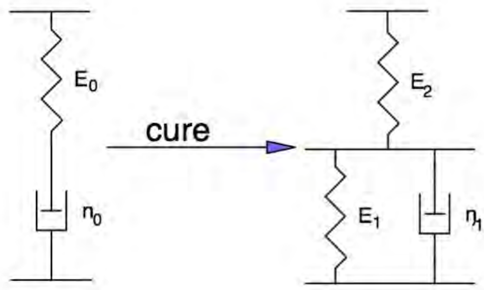

Figure 2 Methacrylate adhesive curing model

The complex modulus of the liquid $G_L(s)$ at the start is

$$G_L(s) = \frac{E_0 \eta_0 s}{E_0 + \eta_0 s} \tag{3}$$

The complex modulus of the solid $G_S(s)$ at full cure is

$$G_S(s) = \frac{E_2(E_1 + \eta_1 s)}{E_2 + E_1 + \eta_1 s} \tag{4}$$

The transition from liquid to solid can then be described using a transition function as

$$H_T(s) = \frac{G_S(s)}{G_L(s)} = \frac{E_2(E_1 + \eta_1 s)(E_0 + \eta_0 s)}{(E_2 + E_1 + \eta_1 s) E_0 \eta_0 s} \tag{5}$$

The complex modulus of the intermediate state $G_\beta(s)$ as the transition progresses can be described by

$$G_\beta(s) = \left[\frac{G_S(s)}{G_L(s)}\right]^\beta G_L(s) = [G_S(s)]^\beta [G_L(s)]^{1-\beta} \tag{6}$$

Testing and Results

Testing of the adhesive was performed using a squeeze film rheometer using 25mm diameter parallel steel plates with a bond gap of 100mm. The magnitude and phase responses of the adhesive were recorded at 0, 20, 40 and 60 minutes and are shown below, along with simulated values for the fractional model based on Equation 6.

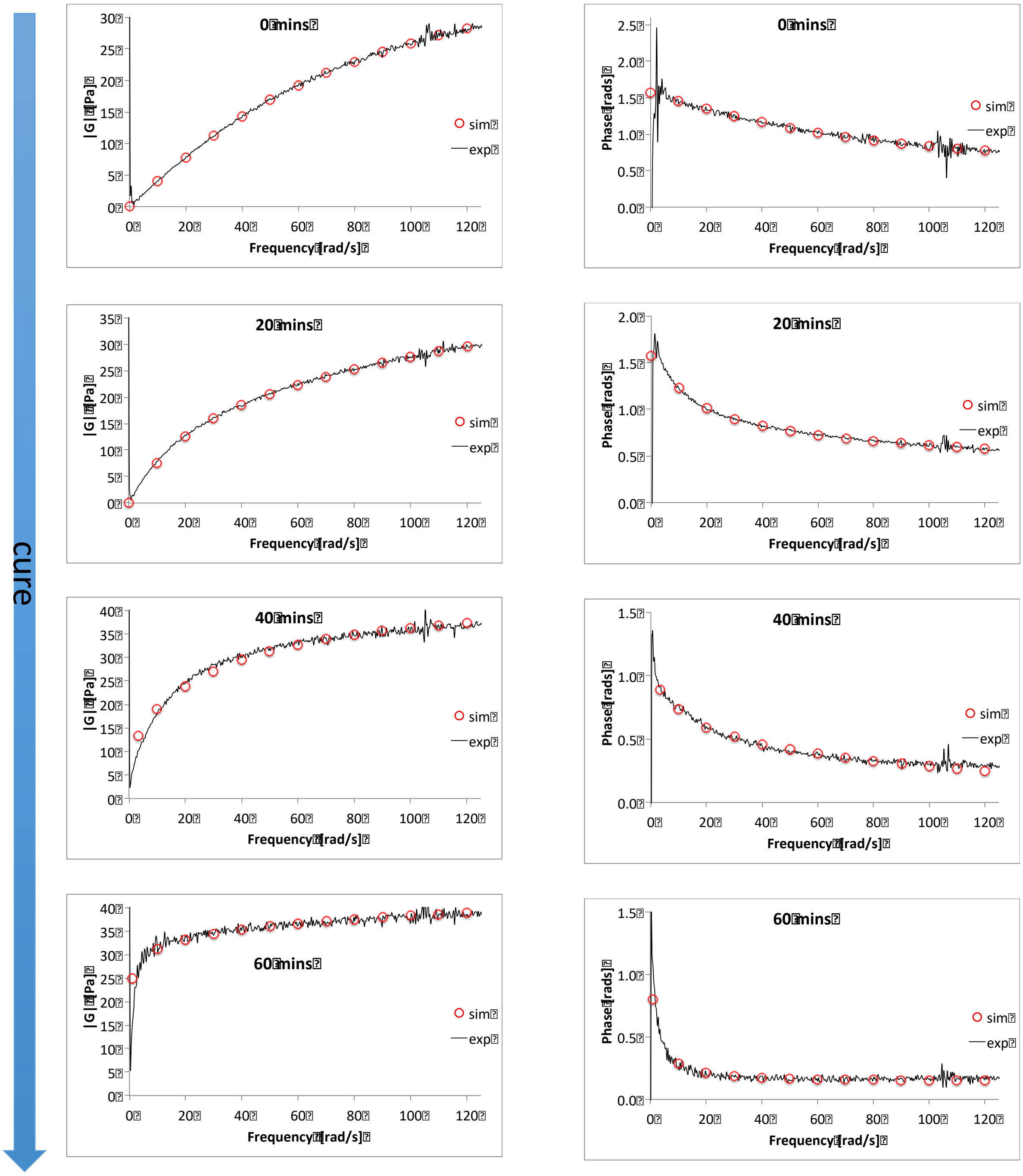

Figure 3 Magnitude and phase plots of methacrylate adhesive as cure progresses

Table I Parameter values for curing plots in Figure 3

Cure time	0 mins	20 mins	40 mins	60 mins
E0 [Pa]	72.4	70.0	38.1	42.6
η0 [Pas]	0.098	0.229	0.384	0.001
E1 [Pa]			4.16	4.03
η1 [Pas]	0.777	2.14	2.19	9.79
E2 [Pa]	48.0	33.3	9.79	15.9
β	0.682	0.581	0.673	0.912

The spectra obtained are based on single spectra at each time period since ensemble averaging is not appropriate for systems that are time varying. However, the results are relatively noise free with a good fit at all times indicating the suitability of the modelling technique.

From Table I, $E_I = 0$ at 0 and 20 minutes indicating that the curing process at these times is best modelled as a transition from one Maxwell system to another. The general characteristic at these times is one of a Maxwell system with high stiffness and low viscosity moving towards a system with lower stiffness and higher viscosity. This can be understood by considering the polymerisation process where polymer chains lengthen rather than crosslink, lowering the stiffness while at the same time reducing the free space in the matrix thereby increasing the shear stress and the effective viscosity.

At 40 and 60 minutes $E_I \neq 0$ and the cure is modelled by a transition from a Maxwell to a SLS system and the magnitude plots show non-zero values at 0 rad/s indicating that solid-like behaviour is present. At 60 minutes the value for β is greater than 0.9 indicating that the solidification has occurred to a large extent. Depending on the type of adhesive and the curing conditions β may not necessarily attain a value of unity. It is at the later stage of cure that the methacrylate adhesive exhibits a flat phase readily associated with fractional derivative behaviour.

Conclusion

The complex modulus of curing adhesive can be described using a fractional derivative model. The principal novelty in this paper lies in a systematic approach that allows one to develop a fractional derivative model from the basis of well-understood standard integer order models and in turn allows for a physical understanding of the curing process.

The fractional power will depend on how the system is restructuring and the stage of the restructuring and is therefore dependent on topology and time. The method used to establish the fractional model is capable of accurately predicting both the amplitude and phase behaviour of phase transition and should be an appropriate method for modelling change of complex modulus in any process that does not exhibit instantaneous phase change.

The methodology consists of three steps as follows:

1. Choose complex models for the start and end points of the transition that are based on standard models of integer powers in s.
2. Formulate the transition function based on these boundary models.
3. Calculate the interstitial states of the complex modulus by multiplying the initial state by a fractional power of the transition function.

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A Note on Optimal Discretization of Fractional Order Filter Functions

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Abstract: As a result of ongoing progresses in digital microelectronics technology, digital systems have played a central role in day life. Therefore, the digital filter design and implementation have turned into a central topic of signal processing. The discrete-time filter functions have been widely utilized in implementation and simulation of the linear time invariant (LTI) system models. For implementation of fractional order filter functions in digital systems, discrete filter functions, which approximate to the frequency response of fractional order transfer functions, are widely used. A major problem, which arises in this digital filter realization, is the frequency response matching performance and the stability of the resulting digital filters. This extended abstract presents a brief review of the book chapter that addresses the problem of optimal and stable IIR filter implementation of fractional order filter functions. In the study, an optimization problem is introduced to find out the optimal coefficients of a stable IIR filter function, where the amplitude response of IIR filter function approximates the amplitude response of original fractional order filter function in an operating frequency range. The finding of this study may contribute to the solution of optimality and stability problems of discrete realization of fractional order filter functions.

Extended Abstract

This extended abstract presents an optimal implementation method for fractional order filters in the form of stable IIR filters. The discussion in this paper was originally published in a book chapter with the title of *Mathematical Methods in Engineering* [2]. This chapter contributes to research works related Task 1.7 of Cost Action CA15225 that is "*Investigation of preservation of properties of non-integer order control and dynamical systems, characterizations and algorithms under discretization*" [1]. Due to obtaining improved frequency selectivity by using fractional order filter function, fractional order filter functions have been implemented in digital systems [3].

In general, filters are used to obtain desired frequency selectivity by configuring amplitude response of filters. They are mainly designed by shaping three types of characteristic regions that are pass bands, stop bands and transition bands. Mainly, stop bands are configured to reject frequency components of undesired signal from the original information signal. Stop band performance of filters is particularly important for signal filtering applications for instance noise signals and harmonic distortions etc.

We focused on the design problem of the stable IIR discrete filter that represents frequency selectivity properties of fractional-order filters, particularly at stop bands [2]. Many analytical discretization methods do not ensure stability of resulting IIR filters, and unstable filter solutions are commonly useless in practice. To deal with the stability problem in filter discretization and improve amplitude response fitting to fractional-order filter in desired frequency ranges, heuristic optimization method have been utilized [2]: The PSO algorithm was modified to obtain stable IIR filter coefficients that are approximating to amplitude responses of fractional-order continuous filter functions. During PSO optimization, the stability of resulting IIR discrete filters is guaranteed by setting very high cost values to the solution particles that result in unstable IIR discrete filter solutions [2]. Thus, solution particles in the swarm are forced to move towards search regions, where stable IIR filter solutions can exist. Illustrative design examples are shown to evaluate performance of the proposed method and results are compared with the results of CFE approximation method.

Let's assume that the fractional-order continuous LTI filters can be written by

$$F_c(s) = \frac{\sum_{i=0}^{k} c_i s^{\beta_i}}{\sum_{i=0}^{m} d_i s^{\alpha_i}} \quad . \tag{1}$$

By considering $s^{\alpha}=(j\omega)^{\alpha}=\omega^{\alpha}(\cos(\alpha\pi/2)+j\sin(\alpha\pi/2))$, the amplitude response of fractional-order filters are expressed as

$$|F_c(j\omega)| = \frac{\left|\sum_{i=0}^{k} c_i \omega^{\beta_i} (\cos(\frac{\pi}{2}\beta_i) + j\sin(\frac{\pi}{2}\beta_i))\right|}{\left|\sum_{i=0}^{m} d_i \omega^{\alpha_i} (\cos(\frac{\pi}{2}\alpha_i) + j\sin(\frac{\pi}{2}\alpha_i))\right|}, \tag{2}$$

where filter coefficients are denoted by $\{c_i, d_i\}$ and fractional orders are represented by the parameters $\{\beta_i, \alpha_i\}$ that are real numbers [2]. This provides more options in frequency selectivity for filter functions compared to integer order counterparts that allow only integer numbers for the order parameters [2]. For digital realization of the amplitude response of fractional filters, one needs the discretization of filter function. In this study, an optimization problem is defined to perform approximation of amplitude responses of an initial randomly generated discrete IIR filter solution set to amplitude response of the continuous fractional-order filter function $F_c(s)$. The discrete IIR filter $F_d(s)$ to implement $F_c(s)$ function is expressed in general form as

$$F_d(z) = \frac{\sum_{i=0}^{l} a_i z^i}{\sum_{i=0}^{p} b_i z^i} \quad . \tag{3}$$

This optimization function is defined as:

$$f(a,b) = \frac{1}{L} \sum_{\omega_i \in (\omega_{\min}, \omega_{\max})} \left(20\log_{10}|F_c(j\omega_i)| - 20\log_{10}|F_d(a,b,e^{j\omega_i T_s})|\right)^2, \tag{4}$$

where $a=[a_0\ a_1\ a_2\ a_3\ ...\ a_l]$ and $b=[b_0\ b_1\ b_2\ b_3\ ...\ b_p]$ are IIR filter coefficients to be optimized in the sampled frequency range $\omega_i \in (\omega_{\min}, \omega_{\max})$, $i=1, 2, 3, ... L$. To calculate Eq. 4, a $z=e^{j\omega Ts}$ mapping is used in Eq. 3. Parameter T_s denotes the sampling period of the discrete filter.

This optimization problem is solved by the PSO algorithm. Positions of solution particles in the coefficient search space are expressed as [2]

$$x_n = [b_0\ b_1\ b_2\ b_3\ ...\ b_p\ a_0\ a_1\ a_2\ a_3\ ...\ a_l]. \tag{5}$$

Particles of PSO move in the search space of filter coefficients. Positions of each particle represent IIR filter solutions in the search space. During the optimization process of PSO algorithm, particles move to minimize the objective function (Eq. 4). A decrease in values of objective functions infers better approximation of amplitude response of $F_d(s)$ filter function to amplitude response of $F_c(s)$ filter function. The following two assets are utilized in design of this objective function for filter application:

(i) Similarity of amplitude response is expressed by mean square of amplitude response difference in logarithmic scale.

(ii) Stability prevention is accomplished by assigning high value to the objective function in case of the existence of unstable filter solution: To achieve the stability of IIR filters, we set a very high cost value (f_{max}) to the objective function for particle solutions if they result in unstable IIR filter solutions. The objective function with this stability constraint can be expressed as

$$f(a,b) = \begin{cases} \frac{1}{L} \sum_{\omega_i \in (\omega_{min}, \omega_{max})} \left(20\log_{10}\left|F_c(j\omega_i)\right| - 20\log_{10}\left|F_d(a,b,e^{j\omega_i T_s})\right|\right)^2 & stable \\ f_{max} & unstable \end{cases} . \quad (6)$$

Here, very high value setting to f_{max} leads to keeping particles away from the region of search space where unstable filter solutions are emerged [2,3]. This enforces particles to search for stable IIR filter solutions. Steps of the proposed PSO algorithm can be summarized as follows,

Step 1: Set initial values to position and velocity of particles randomly.
Step 2: Find the local best and global best positions according to objective function (Eq. 6).
Step 3: Update particle positions according to the position and velocity update procedure of PSO, and determine local best and global best positions by the objective function (Eq. 6).
Step 4: If the maximum number of iterations is reached, stop the optimization. Otherwise, go to Step 3.

For an illustrative fractional order filter discretization, we designed a stable IIR filter that can approximate to the continuous fractional-order Chebyshev low pass filters [4]

$$F_{c2}(s) = \frac{a_0}{a_1 s^{1+\alpha} + a_2 s^{\alpha} + 1} \quad (7)$$

for $\alpha = 0.2$, $a_0 = 3$, $a_1 = 3$ and $a_2 = 5$. After 1000 iteration, the optimized IIR filter function was obtained as

$$F_{d2}(s) = \frac{-0.006286\,z^4 - 0.004119\,z^3 - 0.01841\,z^2 + 0.02865\,z - 0.009169}{19.95z^4 - 4.422\,z^3 - 15.4z^2 + 2.839\,z - 2.908} . \quad (8)$$

The IIR filter function, that was obtained by CFE+Tustin method, is found

$$F_{cfe2}(s) = \frac{0.0007416\,z^9 - 0.004903\,z^8 - 0.01313\,z^7 - 0.0169\,z^6 + 0.006652\,z^5 + 0.01021\,z^4 - 0.0169\,z^3 + 0.0111\,z^2 - 0.00363\,z + 0.0004868}{z^9 - 8.666z^8 + 33.36z^7 - 74.9z^6 + 108z^5 - 103.9z^4 + 66.54z^3 - 27.39z^2 + 6.567z - 0.7009} . \quad (9)$$

Fig. 1 shows evolution of the objective function value during optimization by using the PSO algorithm. The figure indicates the convergence of the optimization process. Fig. 2 compares the amplitude responses of continuous fractional-order filter function $F_{c2}(s)$, the proposed IIR filter function $F_{d2}(s)$, the continuous integer order filter function approximation by CFE method and the discrete IIR filter function approximation by CFE with Tustin method. As seen in figures, the proposed PSO algorithm yields better approximation at stop band of the fractional-order filter function. However, the CFE method can provide superior approximation at low frequency regions. Table 1 lists Mean square error performance of the proposed discrete filter designs by PSO and the filter by using CFE+Tustin method. Fig. 3 shows time response of discrete $F_{d2}(s)$ and $F_{cfe2}(s)$ filter functions for sinusoidal input signal ($\sin(40t)$). Results reveal that $F_{cfe2}(s)$ filter is not stable because CFE method does not ensure the stability of resulting filters. The proposed method performs in the search space that results in stable filters and this is an important advantage for stable discrete filter implementations [2].

Table 1 Mean squared errors for amplitude responses of IIR filter designs [2]

IIR Filter Design Methods	Mean Squared Errors
PSO	0.2627
CFE+Tustin discretization	76.2223

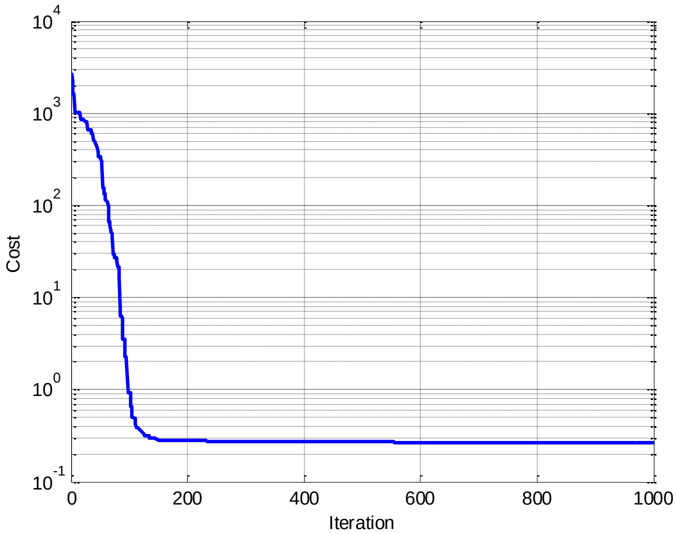

Fig. 1 The objective function values during the optimization of the filter coefficients [2]

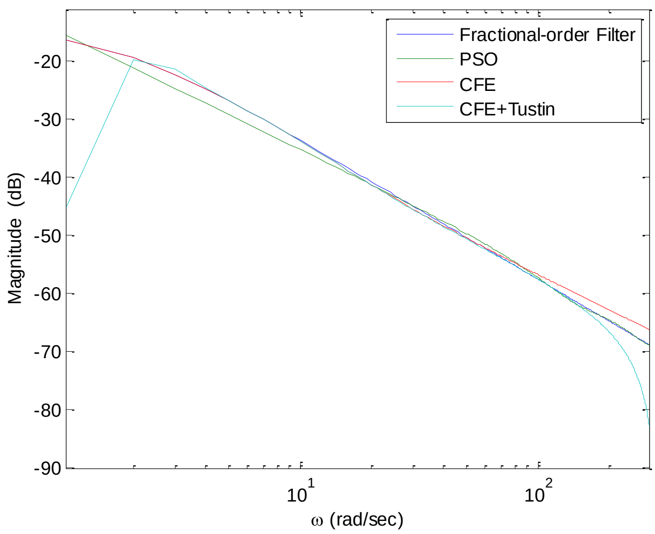

Fig. 2 Comparison of amplitude responses of original fractional-order filter, the IIR filter implementation by using PSO, the continuous integer order filter approximation by CFE and the discrete IIR filter function approximation by CFE+Tustin methods [2]

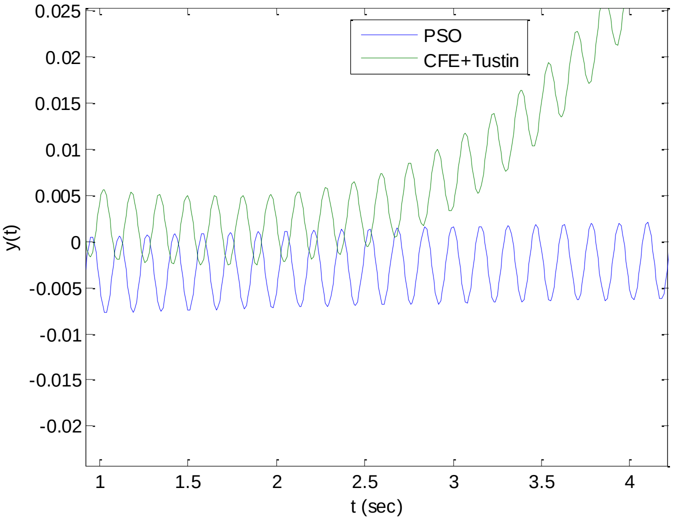

Fig. 3 Time response of discrete filter functions for sinusoidal input signal [2]

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Synthesis of fractional-order elements

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Abstract: While designing fractional-order analog signal processing blocks, a discrete Fractional-Order Element (FOE) introducing the fractional feature of the final function block is required. In the open literature, the FOEs may also be referred to as Elements with Fractional-Order Impedance (EFI), Constant-Phase Elements (CPE), or simply Fractors [1].

A recent summary [2] shows a number of prospective technologies being investigated to introduce capacitive FOEs. One of these prospective design technologies, that was further investigated with the COST Action CA15225 is the design based on the using homogenous distributed resistive-capacitive (RC) structures (lines). A software tool was designed suitable for the synthesis of capacitive FOEs as we described in [3], whereas using a thick-film technology discrete FOEs were also experimentally implemented.

Keywords: solid-state fractional-order element; RC network with distributed parameters; thick-film technology

Extended abstract

The relation between cross voltage and through current of capacitive and inductive FOE (FO capacitor and FO inductor) can be represented by fractional differentiation as follows [4]:

$$i_{C_\alpha} = C_\alpha \frac{\mathrm{d}^\alpha v_{C_\alpha}}{\mathrm{d}t^\alpha}, \quad (1)$$

$$v_{L_\beta} = L_\beta \frac{\mathrm{d}^\beta i_{L_\beta}}{\mathrm{d}t^\beta}, \quad (2)$$

where α and β are the fractional orders of the fractional capacitor and inductor in the range (0; 1), respectively. By Laplace transformation of (1) and (2), the impedance of these elements have the form:

$$Z_{C_\alpha}(s) = \frac{1}{s^\alpha C_\alpha}, \quad (3)$$

$$Z_{L_\beta}(s) = s^\beta L_\beta, \quad (4)$$

where s is the Laplace operator (complex frequency), the constants C_α and L_β are also referred to as pseudo-capacitance and pseudo-inductance having units $\mathrm{Fs}^{\alpha-1}$ and $\mathrm{Hs}^{\beta-1}$, respectively. Substituting $s = \mathrm{j}\omega$ into (3) and (4) we may observe that the phase of the impedance of FO capacitor equals $-\alpha\pi/2$, whereas the phase of the impedance of FO inductor equals $\beta\pi/2$. Hence, the phase of FOE is constant and independent of frequency.

While designing classic (integer-order) analog circuits, capacitors are commonly preferred to inductors. Similarly, more attention is also given to FO capacitors than FO inductors, as it can also be seen from the survey [2]. Therefore, within our research activities we also focused on the design of capacitive FOEs by taking advantage of resistive-capacitive (RC) circuits with distributed parameters.

The idea of realizing impedances with given characteristics by RC circuits with distributed parameters started to be discussed already more than 60 years ago, see e.g. [5], [6]. Here, the synthesis methods were based on utilizing homogenous RC lines of the form R-C-0 (resistor-capacitor-conductor). Based on our further analyses in this area (see e.g. [7], [8], [9]), the R-C-NR layer structure shows to be very suitable for efficient design of FOEs. Such structure contains two resistive layers with resistances R and $N \cdot R$, and one capacitive (dielectric) layer with capacitance C between these two resistive layers. The resistive layers are bonded and hence the R-C-NR structure represents a 4-terminal circuit as shown in Fig. 1.

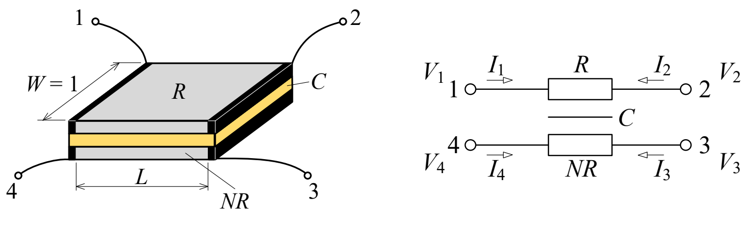

Figure 1: (a) The 3D view of R-C-NR structure, (b) its equivalent schematic

To achieve fractional order of the designed FOEs different to the value of 0.5, we have developed a structural-parametric synthesis method of FOE, which is in detail described in [3]. The design tool employing genetic algorithms assumes four four-terminal R-C-NR structures-sections and as an output it provides the interconnection of individual sections (Fig. 2), defines their geometry, i.e. their lengths L (relative to unity width $W = 1$), layer resistivity R, layer capacitance C, and the ratio of the resistivity of the top and bottom resistive layers N to provide at the input an impedance Z_{IN} with required phase shift in the required frequency band.

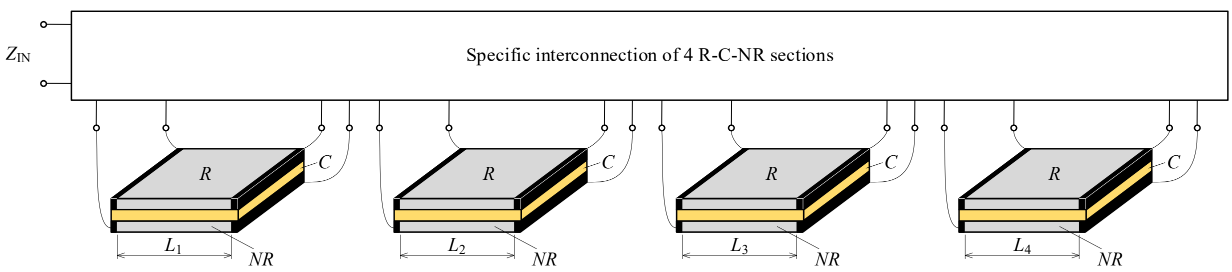

Figure 2: Simplified view on the result of the design tool specifying the parameters of individual R-C-NR sections and their interconnection

To show and validate the possibilities of FOE design based on the approach of RC circuits with distributed parameters, using the outputs of the design tool FOEs were experimentally implemented using the thick-film technology [10], [11]. In Fig. 3, the top view on the realized capacitive FOE is shown, where the individual R-C-NR sections can also be identified. This capacitive FOE is primarily characteristic with its fractional order $\alpha = 0.45$ and pseudo-capacitance $C_\alpha = 120\ \text{nFs}^{-0.55}$ (Fig. 4), and is being labeled as capFOE_045. The fractional feature (the constant phase region) in the frequency band from 8 kHz to 4 MHz. The dimension of the designed capFOE_045 is 42 mm × 13 mm (excluding pins). The overview of other main parameters of the capFOE_045 is listed in Table 1.

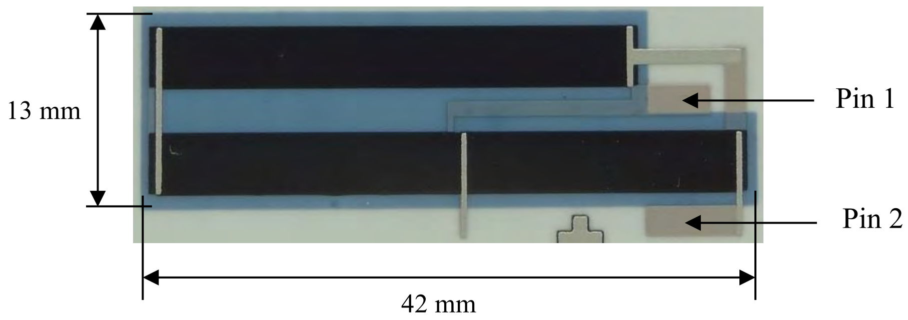

Figure 3: A sample of capacitive FOE designed in thick-film technology

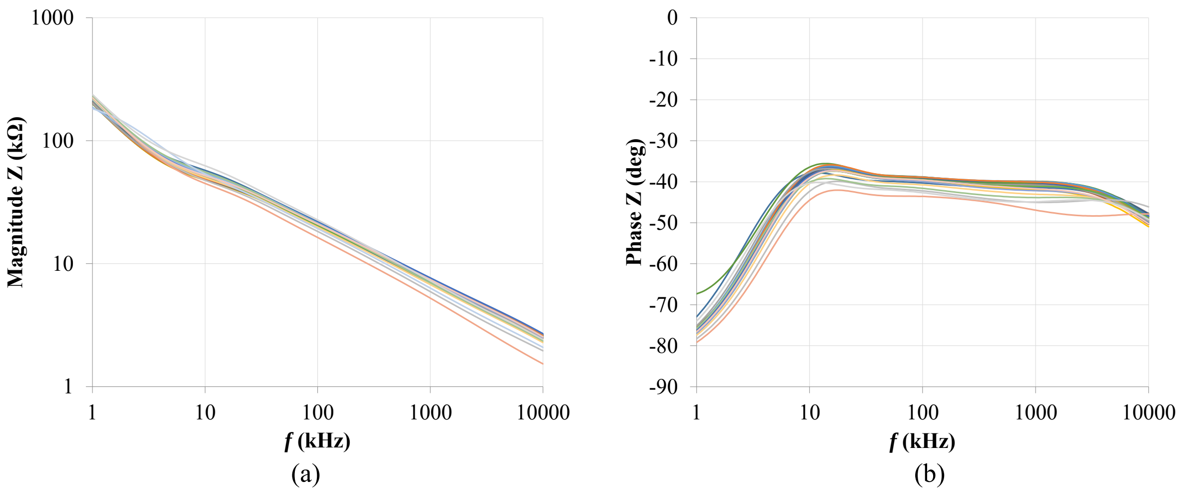

Figure 4: Impedance magnitude and phase of capFOE_45 (26 samples measured)

Table 1: Electrical characteristics of capFOE_045

Parameter	Symbol	Min	Typ	Max	Unit
Fractional order	α	0.44	0.45	0.49	-
Fractance	C_α	68	120	142	$nFs^{-0.55}$
Min. frequency of operation	f_{min}		8		kHz
Max. frequency of operation	f_{max}		4		MHz
Impedance magnitude[1]			15.2		kΩ
Impedance phase[1]			-40.6		deg
Max. absolute error in impedance magnitude[1]				3.2	kΩ
Max. absolute error in impedance phase[1]				3.5	deg
Max. relative error in impedance magnitude[1]				21	%
Max. relative error in impedance phase[1]				8.6	%

[1]at central frequency 180 kHz

Conclusion and Future Work

The design of capacitive FOEs as solid-state elements using the theory of RC circuits with distributed parameters shows to be feasible. So far, the thick-film technology was used for experimental realization. However, based on our preliminary analyses, the think-film technology may also be used. For this purpose, the technology process needs further investigation to be able to implement the individual layers with specific parameters (resistivity and capacitance).

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Approximation of fractional-order blocks

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Abstract: Fractional-order systems and controllers are based on irrational operators or transfer functions that require an approximation by rational transfer functions. Namely, the approximation allows realization of such fractional-order elements. This extended abstract synthesizes some results that were obtained, in the period of the COST Action CA15225, namely suitable and efficient approximation methods. Remarkable results regard the symmetry of the zero-pole patterns in discretization of the fractional-order Tustin operator and the simple second-order realization of a fractional-order lead compensator. The links established with researchers from countries in the COST Action made some of these results possible.

Keywords: fractional-order operator; approximation; fractional-order lead compensator.

Extended abstract

This abstract describes some results useful to approximate irrational fractional-order operators that can be used as building blocks of many fractional-order systems and controllers. Other illustrated results allow to approximate the irrational transfer function of a fractional-order lead compensator that is the basic element of a recently introduced class of controllers.

Many fractional-order circuits and systems, and fractional-order controllers, are based on a fractional-order operator (or basic block) that is the irrational differentiator or integrator s^{ν}, with $\nu > 0$ (differentiator) or $\nu < 0$ (integrator). This operator as well as other irrational fractional-order blocks require an approximation specified by a rational transfer function (also named the "approximant") in a frequency range of interest. The approximant must enjoy stability and minimum-phase properties, i.e. its poles and zeros must lie in the left-half of the Gauss plane (analog realization) or inside the unit circle (discrete realization). Moreover, if poles and zeros are simple and alternate on the negative real half-axis, thus enjoying the "interlacing" property, the approximant is a positive real function [1] and s^{ν} can be realized by a network with linear and passive elements only [2].

Truncation of Continued Fraction Expansions (CFEs) is very useful because it can directly establish the values of the electrical elements of analog circuit realizations. The main realizations use simple passive impedances referring to truncated CFEs of Stieltjes's type (S-CFEs). Positive signs of the constants in the partial denominators of S-CFEs are conditions to guarantee stable and minimum-phase realizations with real zeros interlacing real poles. These properties of the truncated S-CFEs with positive coefficients are classical in analog and discrete realizations.

The literature offers many other CFEs for approximating s^{ν} and the digital counterpart, some of which maintain the stability, minimum-phase, and interlacing properties. However, there was no rigorous and complete proof that these CFEs enjoy such properties for any ν, with $0 < \nu < 1$, and for any degree n of the numerator and denominator polynomials in the rational transfer function. Moreover, there exist no direct formulas to convert other forms of CFEs into S-CFEs. Hence, it is impossible to use a direct conversion as a means to determine the zeros and poles of a given truncated CFE.

In [3], the classical Lagrange's CFE (L-CFE)

$$(1+x)^{\nu} = 1 + \frac{a_1\,x}{1+\frac{a_2\,x}{1+\frac{a_3\,x}{1+\cdots}}}, \tag{1}$$

with $a_1 = \nu$, $a_j = \frac{i-\nu}{2\,(2i-1)}$, $a_{j+1} = \frac{i+\nu}{2\,(2i+1)}$, $j = 2i$, $i \geq 1$, is revisited as a S-CFE, namely it holds $a_1 > 0$, $a_j > 0$, $a_{j+1} > 0$. Then, any CFE equivalent to (1) is an S-CFE. Moreover, truncation of (1) provides the approximation which can be reduced to a rational transfer function

$$G_n(x) = \frac{\alpha_{n,0}\, x^n + \alpha_{n,1}\, x^{n-1} + \alpha_{n,2}\, x^{n-2} + \cdots + \alpha_{n,n-2}\, x^2 + \alpha_{n,n-1}\, x + \alpha_{n,n}}{\beta_{n,0}\, x^n + \beta_{n,1}\, x^{n-1} + \beta_{n,2}\, x^{n-2} + \cdots + \beta_{n,n-2}\, x^2 + \beta_{n,n-1}\, x + \beta_{n,n}}, \tag{2}$$

where the author showed that coefficients α's and β's can be computed by closed-form expressions as follows:

$$\begin{aligned} \alpha_{n,k} &= \frac{(k+1)_n}{(n-k)!}\, (k+1+\nu)_{(n-k)} \\ \beta_{n,k} &= \frac{(k+1)_n}{(n-k)!}\, (k+1-\nu)_{(n-k)} \end{aligned} \tag{3}$$

for $k = 0, 1, 2, \dots, n$, where $(k+1 \pm \nu)_{(n-k)} = (k+1 \pm \nu)(k+2 \pm \nu) \cdots (n \pm \nu)$ are Pochhammer symbols finishing with $(n+1 \pm \nu)_{(0)} = 1$ when $k = n$, and $(k+1)_n = (k+1)(k+2)(k+2) = \frac{(k+n)!}{k!}$.

Then a link between the L-CFE and two classes of CFEs is established, such that the position of poles and zeros of the approximants can be determined. Stability, minimum-phase and interlacing between stable poles and minimum-phase zeros are proven for any $0 < \nu < 1$ and for any value of n. This result is obtained both in the s-domain (for analog realization), with zero-pole interlacing along the negative real half-axis, and in the z-domain (for discrete realization), in which case real zeros and real poles are placed inside the unit circle. Moreover, it is proven that the polynomial coefficients in the discrete approximation enjoy such properties that poles and zeros are symmetrical.

Truncation of (1) to the last partial denominator $a_{2n}\, x$ and substitution $x = s - 1$ provides a first class of approximations

$$M_n(s) = \frac{p_{n,0}\, s^n + p_{n,1}\, s^{n-1} + p_{n,2}\, s^{n-2} + \cdots + p_{n,n-2}\, s^2 + p_{n,n-1}\, s + p_{n,n}}{q_{n,0}\, s^n + q_{n,1}\, s^{n-1} + q_{n,2}\, s^{n-2} + \cdots + q_{n,n-2}\, s^2 + q_{n,n-1}\, s + q_{n,n}} \tag{4}$$

where coefficients p's and q's can be computed by closed-form expressions given in [4]-[5].

Another approximation

$$T_n(s) = \frac{\overline{p}_{n,0}\, s^n + \overline{p}_{n,1}\, s^{n-1} + \overline{p}_{n,2}\, s^{n-2} + \cdots + \overline{p}_{n,n-2}\, s^2 + \overline{p}_{n,n-1}\, s + \overline{p}_{n,n}}{\overline{q}_{n,0}\, s^n + \overline{q}_{n,1}\, s^{n-1} + \overline{q}_{n,2}\, s^{n-2} + \cdots + \overline{q}_{n,n-2}\, s^2 + \overline{q}_{n,n-1}\, s + \overline{q}_{n,n}} \tag{5}$$

is based on computing ω^ν and its first n derivatives with respect to ω in a central point ω_0, then establishing coincidence of these magnitudes with those given by $T_n(s)$ in the ω-domain. The approximation is derived by truncating the CFE

$$d_0 + \frac{s-\omega_0}{d_1 + \frac{s-\omega_0}{d_2 + \frac{s-\omega_0}{d_3 + \cdots}}}, \tag{6}$$

and stopping at d_{2n}, where the coefficients are computed as $d_0 = \omega_0^\nu$, $d_1 = \omega_0^{1-\nu}/\nu$, $d_{2i} = \frac{2\nu(1+\nu)(2+\nu)\cdots(i-1+\nu)}{(1-\nu)(2-\nu)\cdots(i-\nu)}\, \omega_0^\nu$, $d_{2i+1} = \frac{(2i+1)(1-\nu)(2-\nu)\cdots(i-\nu)}{\nu(1+\nu)(2+\nu)\cdots(i+\nu)}\, \omega_0^{1-\nu}$. In [3], it is shown that, for $\omega_0 = 1$, $M_n(s) = T_n(s)$, while, for $\omega_0 \neq 1$, $\omega_0^\nu\, M_n\left(\frac{s}{\omega_0}\right) = T_n(s)$.

The main result was proving that numerator and denominator polynomials form a *positive pair*, i.e. their roots are negative real, simple, and alternating along the negative real half-axis. However, considering the CFEs leading to $M_n(s)$ and $T_n(s)$, it is hard to reduce them to the S-CFE, determine the coefficients in the S-CFE as nonlinear functions of ν, and find the conditions on ν to verify positiveness of coefficients. Then, the proof given in [2] is based on the relation between the L-CFE and the developed approximation, hence between the numerator and denominator of $G_n(x)$ and those of $M_n(s)$ and $T_n(s)$. In particular, the zeros $(-\sigma_i)$ and poles $(-\tau_i)$ of $M_n(s)$ are related by $\sigma_i = 1/\tau_{n-i+1}$, for $i = 1, \dots, n$.

Another important result in [3] was giving formulas for the coefficients of the approximation in the discrete domain and proving that zeros and poles are interlaced inside the unit circle. The approximants of the Tustin discrete fractional-order operator are expressed by

$$\left(\frac{2}{T}\right)^{\nu} H_n(z) = \left(\frac{2}{T}\right)^{\nu} \frac{c_{n,0}\, z^n + c_{n,1}\, z^{n-1} + c_{n,2}\, z^{n-2} + \cdots + c_{n,n-2}\, z^2 + c_{n,n-1}\, z + c_{n,n}}{d_{n,0}\, z^n + d_{n,1}\, z^{n-1} + d_{n,2}\, z^{n-2} + \cdots + d_{n,n-2}\, z^2 + d_{n,n-1}\, z + d_{n,n}} \tag{7}$$

where

$$\begin{aligned} c_{n,i} &= (n-i+1)_n \sum_{j=0}^{i} \frac{(-2)^{i-j}}{(i-j)!\,j!} (2n-i+1)_{(j)}\, (n-i+j+1+\nu)_{(j)} \\ d_{n,i} &= (n-i+1)_n \sum_{j=0}^{i} \frac{(-2)^{i-j}}{(i-j)!\,j!} (2n-i+1)_{(j)}\, (n-i+j+1-\nu)_{(j)} \end{aligned} \tag{8}$$

Moreover, it was proven that poles and zeros of $H_n(z)$ are real, strictly interlaced, located between -1 and +1 in the z-plane, and enjoy symmetry with respect to the origin.

An example regarding the approximation of $s^{0.6}$ with $n = 4$ is given in Table 1. Zero-pole interlacing can be verified in all domains.

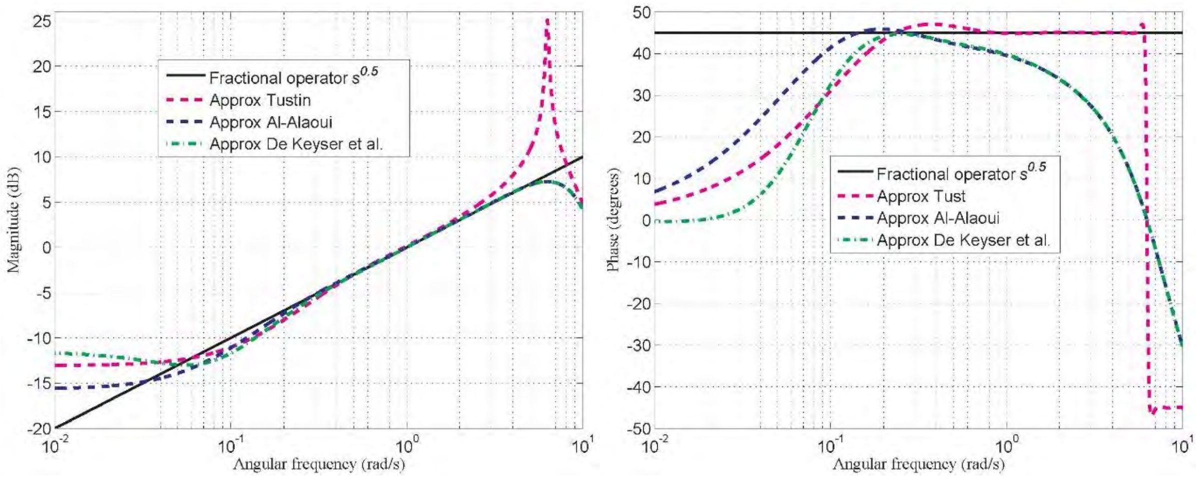
Table 1: Coefficients, zeros, and poles of $G_4(x)$, $M_4(s)$, $H_4(z)$ for $\nu = 0.6$

$G_4(x)$	$\alpha_{4,0}$	$\alpha_{4,1}$	$\alpha_{4,2}$	$\alpha_{4,3}$	$\alpha_{4,4}$	Zeros			
	68.89	861.12	2980.8	3864	1680	-7.85	-2.32	-1.31	-1.02
	$\beta_{4,0}$	$\beta_{4,1}$	$\beta_{4,2}$	$\beta_{4,3}$	$\beta_{4,4}$	Poles			
	4.57	228.48	1468.8	2856	1680	-42.83	-4.27	-1.76	-1.15
$M_4(s)$	$p_{4,0}$	$p_{4,1}$	$p_{4,2}$	$p_{4,3}$	$p_{4,4}$	Zeros			
	68.89	585.56	810.78	210.20	4.57	-6.85	-1.32	-0.31	-0.02
	$q_{4,0}$	$q_{4,1}$	$q_{4,2}$	$q_{4,3}$	$q_{4,4}$	Poles			
	4.57	210.20	810.78	585.56	68.89	-41.83	-3.27	-0.76	-0.15
$H_4(z)$	$c_{4,0}$	$c_{4,1}$	$c_{4,2}$	$c_{4,3}$	$c_{4,4}$	Zeros			
	1680	-1008	-1180.8	493.44	88.4736	0.95	-0.75	0.53	-0.14
	$d_{4,0}$	$d_{4,1}$	$d_{4,2}$	$d_{4,3}$	$d_{4,4}$	Poles			
	1680	1008	-1180.8	-493.44	88.4736	-0.95	0.75	-0.53	0.14

Note that, to perform discretization, one can apply direct or indirect procedures. However, direct discretization of the fractional-order Tustin operator may lead to an approximant with coefficients having very large or very small values, although interlacing of stable poles and minimum-phase zeros is guaranteed [6]-[7]. This is an unfavorable feature because numerical problems can be determined by overflow and/or underflow of the floating point representation of the coefficients.

Then, in [7], the authors proposed an indirect two-steps approach avoiding numerical problems determined by the values of the coefficients. Moreover, the approximant coefficients are given by closed-form expressions. In the first step of the approach, the fractional binomial function $(1 + x)^{\nu}$ is expressed as rational transfer function with coefficients in closed forms. In the second step, the previous transfer function is converted to a rational discrete transfer function modeling the fractional-order Tustin operator, with coefficients that are also given by formulas in closed form. These expressions enlighten the characteristic symmetric zero-pole patterns. To synthesize, the approach makes closed-form formulas available for the coefficients of rational transfer functions representing the basic fractional-order block, both in the analog and discrete cases. The formulas avoid tedious algebraic operations of the indirect approaches and numerical problems.

For example, the proposed method approximates $s^{0.5}$ by a discrete transfer function with $n = 4$ as follows: poles are in -0.9397, -0.5, 0.1736, 0.7660; zeros are in -0.7660, -0.1736, 0.5, 0.9397. Symmetry is clear. Other band-limited approximations of $s^{0.5}$ are compared [7]. The frequency responses are shown in Fig. 1. The discretization based on the Tustin operator is affected by large errors in the magnitude diagram at high frequencies, while lower errors are obtained by the other schemes. However, the phase diagram of the Tustin approximation is much closer to the constant-phase diagram, which is very important for robust control.



(a) (b)

Figure 1: Frequency response of approximations: (a) magnitude diagram; (b) phase diagram

Another result is the approximation of an irrational fractional-order lead compensator

$$H(s) = \left(\frac{1 + \tau\, s}{1 + \tau\, \Delta\, s}\right)^{\nu}, \tag{9}$$

where $0 < \nu < 1$ is the fractional (non-integer) order, $\tau > 0$, $0 < \Delta < 1$. Control design is illustrated in [8], in which the author also provided the realization method. The starting point is the approximation of x^{ν} by a second-order rational transfer function

$$x^{\nu} \approx \frac{\sum_{i=0}^{2} b_{2-i}\, x^{i}}{\sum_{i=0}^{2} a_{2-i}\, x^{i}}, \tag{10}$$

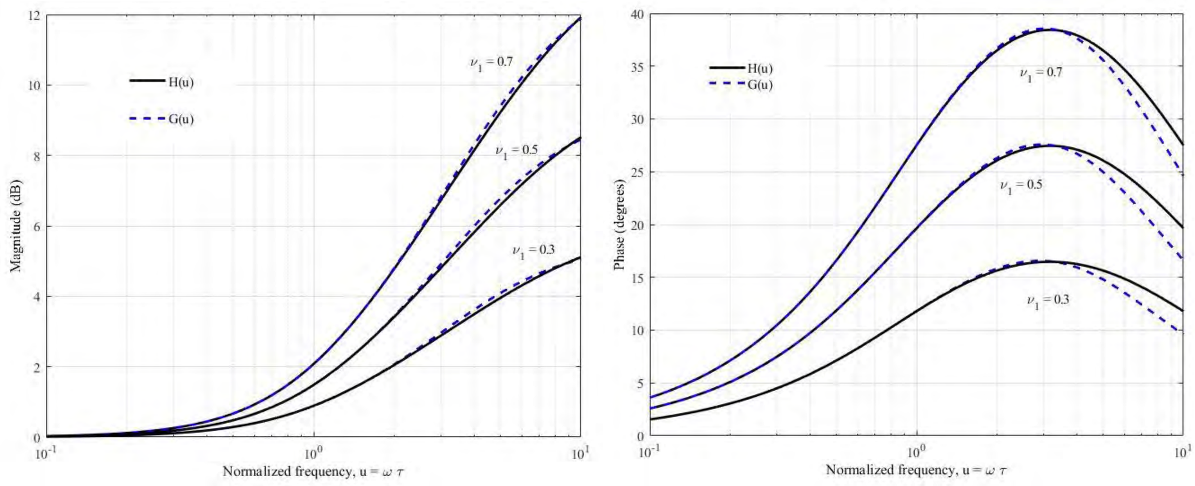
where $b_2 = a_0 = (1 - \nu)(2 - \nu)$, $b_1 = a_1 = 2(2 - \nu)(2 + \nu)$, and $b_0 = a_2 = (1 + \nu)(2 + \nu)$. The transformation $x = \frac{1 + \tau\, s}{1 + \tau\, \Delta\, s}$ converts (10) to the following approximation:

$$\left(\frac{1 + \tau\, s}{1 + \tau\, \Delta\, s}\right)^{\nu} \approx \frac{\sum_{i=0}^{2} b_{2-i}\, (1+\tau\, s)^{i}\, (1+\tau\, \Delta\, s)^{2-i}}{\sum_{i=0}^{2} a_{2-i}\, (1+\tau\, s)^{i}\, (1+\tau\, \Delta\, s)^{2-i}}, \tag{11}$$

which is proved in [8] to be the same as

$$\left(\frac{1 + \tau\, s}{1 + \tau\, \Delta\, s}\right)^{\nu} \approx G(s) = \frac{\sum_{k=0}^{2} B_{2-k}\, s^{k}}{\sum_{k=0}^{2} A_{2-k}\, s^{k}}, \tag{12}$$

with $B_{2-k} = \sum_{i=0}^{2} b_{2-i}\, L_{ki}^{C}$, $A_{2-k} = \sum_{i=0}^{2} a_{2-i}\, L_{ki}^{C}$, $L_{ki}^{C} = \sum_{j=\mu_1}^{\mu_2} \binom{i}{j}\binom{2-i}{k-j}\,\Delta^{k-j}\,\tau^{k}$, $\mu_1 = \max\{0, k+i-2\}$, $\mu_2 = \min\{i,k\}$, $k = 0, 1, 2$. Fig. 2 shows the normalized frequency response of the irrational compensator in (9) with $\Delta = 0.1$, $u = \omega\tau$ (normalized frequency), and $\nu = 0.3, 0.5, 0.7$ (solid lines) and its second-order approximation (dashed lines).



(a) (b)

Figure 2: Normalized frequency response of the fractional-order lead compensator $H(s)$ defined in (9), with fractional order $\nu = 0.3, 0.5, 0.7$, and its approximation $G(s)$: (a) magnitude diagrams (b) phase diagrams

Conclusion and Future Work

Some new methods were proposed for approximation of fractional-order operators and transfer functions in both the analog and discrete domain. Suitable closed-form expressions allow computation of the coefficients of the approximants. Stability, minimum-phase, and interlacing are guaranteed. Peculiar symmetry of zeros and poles of the discrete transfer functions was also obtained. Moreover, a second-order approximant was obtained for a fractional-order lead compensator. Future work can investigate other properties of analog and discrete approximations, for example for high-speed realizations and for control of nonlinear systems.

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Fractional-order analogue filters

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Abstract: Fractional-order filtering is currently a well-known technique for deriving filtering transfer functions that offer a precise control of a slope of the stop band attenuation, in some cases very low cutoff frequencies while using reasonable capacitor values, as well as generation of specific phase shift distance between input and output signals [1]. Current research is focused on the investigation of various types of transfer functions, approximation methods, usability of many types of active elements within the circuit and multifunctionality of designed structure [2-11].

Many of these prospective research topics were investigated within the COST Action CA15225, [2-11], for instance. Most of the works are focused on the fractional-order low-pass (FLPF) or high-pass (FHPF) transfer functions with order less than two, in [3] for instance. However, also band-pass (FBPF) and band-stop (BBSF) responses in fractional-order domain are frequently researched, [8] for example. In some cases, even higher-order fractional-order transfer functions are studied, for instance in [10].

Keywords: approximation method; fractional-order filter; fractional order; reconfigurable filter; transfer function;

Extended abstract

Fractional-order calculus is a very useful tool with many interdisciplinary applications in several areas covering electrical engineering, biology and biomedicine, control systems, and signal processing [1]. The development of fractional-order circuits includes filters, oscillators, biological tissue emulators and analog controllers, published in [2-14], for instance.

One of the common methods of obtaining analogue active filter circuits of a fractional-order ($1+\alpha$), where $0 < \alpha < 1$, is by replacing the classic capacitor(s) in second-order active filter by the fractional-order admittance defined as $Y = s^{\alpha}C$, where $s = j\omega$ is complex variable and $C > 0$ is a constant often referred to as a pseudo-capacitance ($F \cdot s^{\alpha-1}$). This substitution leads to one of the following types of FLPF responses [15]:

$$H_{1+\alpha}^{\text{LP-A}}(s) = \frac{1}{s^{1+\alpha}K_{A1} + s^{\alpha}K_{A2} + K_{A3}}, \tag{1}$$

$$H_{1+\alpha}^{\text{LP-B}}(s) = \frac{1}{s^{1+\alpha}K_{B1} + sK_{B2} + K_{B3}}, \tag{2}$$

$$H_{1+\alpha}^{\text{LP-C}}(s) = \frac{1}{s^{\alpha_1+\alpha_2}K_{C1} + s^{\alpha_2}K_{C2} + K_{C3}}, \tag{3}$$

where the coefficients K_{Ai}, K_{Bi}, or K_{Ci}, ($i = 1, 2, 3$) should be calculated in order to obtain the desired characteristics (bandwidth, pass-band peaking, etc.). There are usually two capacitors in inductor-less integer-order filters (referred to as C_1 and C_2. Case A (1) represents an example of C_1 interchanged by fractional-order counterpart, case B (2) is the case when C_2 is interchanged and case C (3) is when both capacitances are interchanged by fractional-order counterpart. Obtaining the numerical values of these fractional-order transfer function coefficients with the format (1) - (3) stem from minimizing the error between the fractional-order transfer function and the selected integer-order transfer function with a known approximation (Butterworth, Chebyshev, and inverse Chebyshev) or particular value of quality factor [15].

In some cases, such as audio signals processing, fully-differential (F-D) filtering solutions are beneficial. Therefore, these filters are also studied. One example of the designed circuit is shown in Fig. 1 [7]. This particular solution of the filter is multifunctional, i.e. differential FLPF and FBPF are available from the same structure without any need to modify it.

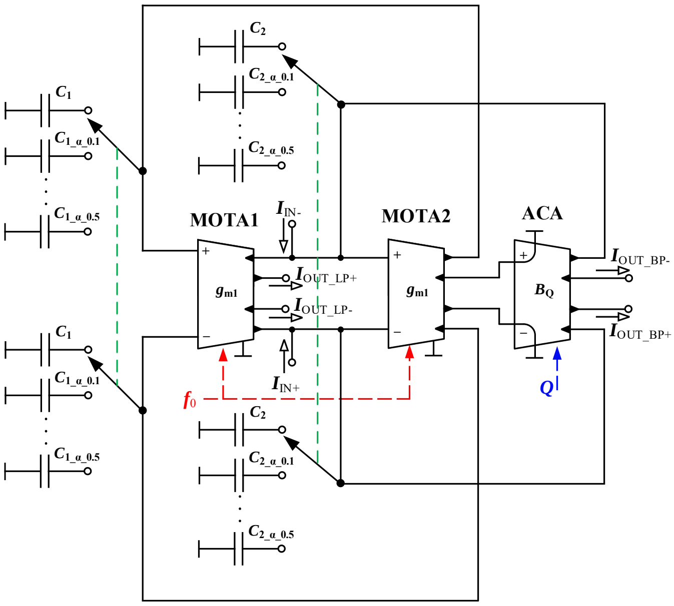

Figure 1: Designed fully-differential (F-D) filtering solution with multiple transfer functions [7]

The transfer functions in the case when C_1 is replaced by fractional-order element (FOE) as follows [7]:

$$TF_{1+\alpha}^{\mathrm{LP}}(s) = \frac{\dfrac{\omega_0^{1+\alpha}}{K_1}}{s^{1+\alpha} + s^{\alpha}\dfrac{K_2\omega_0}{K_1} + \dfrac{K_3\omega_0^{1+\alpha}}{K_1}}, \tag{4}$$

$$TF_{1+\alpha}^{\mathrm{BP}}(s) = \frac{s^{\alpha}\dfrac{\omega_0}{K_1}}{s^{1+\alpha} + s^{\alpha}\dfrac{K_2\omega_0}{K_1} + \dfrac{K_3\omega_0^{1+\alpha}}{K_1}}, \tag{5}$$

Another approach is represented by a reconfigurable filtering structure, which transfer function can be changed without altering the input or output node [10]. These filters are frequently referred to as reconnection-less. Example of such a filter is shown in Fig. 2 [10].

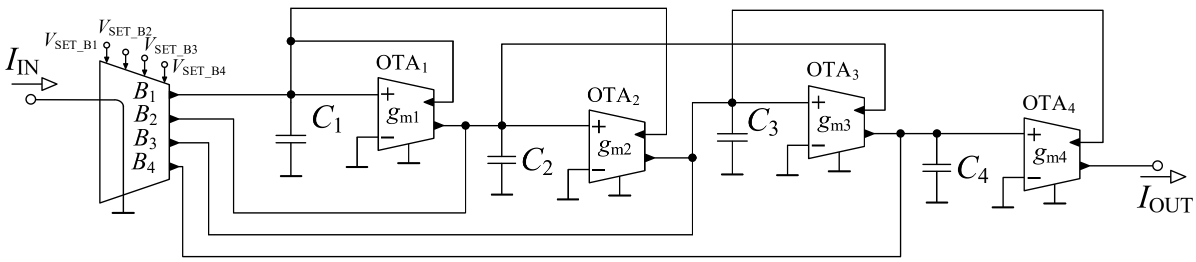

Figure 2: Proposed fractional-order multifunctional filtering topology with FLPF and FHPF responses in the same topology [10]

Integer-order transfer function is equal to [10]:

$$H^{\mathrm{LP}}(s) = \frac{N(s)}{D(s)}, \tag{6}$$

where

$$N(s) = s^3 C_1 C_2 C_3 g_{m4} B_4 + s^2 C_1 C_2 g_{m3} g_{m4} B_3 + s^2 C_2 C_3 g_{m1} g_{m4} B_4 +$$
$$+ s C_3 g_{m1} g_{m2} g_{m4} B_4 + s C_2 g_{m1} g_{m3} g_{m4} B_3 + s C_1 g_{m2} g_{m3} g_{m4} B_4 +$$
$$+ s C_1 g_{m2} g_{m3} g_{m4} B_2 + g_{m1} g_{m2} g_{m3} g_{m4} B_4 + g_{m1} g_{m2} g_{m3} g_{m4} B_2 +$$
$$+ g_{m1} g_{m2} g_{m3} g_{m4} B_1 + g_{m1} g_{m2} g_{m3} g_{m4} B_3 \quad (7)$$

and

$$D(s) = s^4 C_1 C_2 C_3 C_4 + s^3 C_2 C_3 C_4 g_{m1} + s^2 C_3 C_4 g_{m1} g_{m2} +$$
$$+ s^2 C_1 C_4 g_{m2} g_{m3} + s^2 C_1 C_2 g_{m3} g_{m4} + s C_2 g_{m1} g_{m3} g_{m4} + s C_4 g_{m1} g_{m2} g_{m3} + \quad (8)$$
$$+ g_{m1} g_{m2} g_{m3} g_{m4}.$$

Each of the capacitors could be replaced by its fractional-order counterpart. This gradual exchange can offer many types of transfer functions in this case: LP 3+α, LP 2+α, LP 1+α, LP 2+α+β, LP 1+α+β, LP α+β, LP 1+α+β+γ, LP α+β+γ and LP α+β+γ+δ.

Another example of multifunction and very simple fractional-order filter is shown in Fig. 3 [11]. This filtering solution consists of just two PMOS transistors and, therefore, it is MOS only solution. Only two more components are required: current source and fractional-order capacitor (FOC). Two voltage outputs are available, providing FLPF and FHPF response.

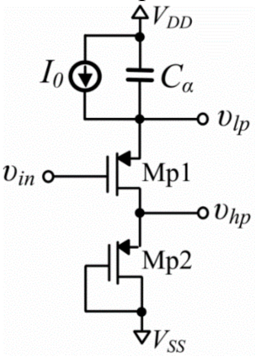

Figure 3: Proposed fractional-order multifunctional filtering topology with FLPF and FHPF responses in the same topology [11]

Transfer functions of this filter are [11]:

$$H_{1+\alpha}^{\mathrm{LP}}(s) = \frac{1}{\left[\left(C_\alpha / g_m\right)^{1/\alpha} s\right]^\alpha + 1}, \quad (9)$$

$$H_{1+\alpha}^{\mathrm{HP}}(s) = \frac{\left[\left(C_\alpha / g_m\right)^{1/\alpha} s\right]^\alpha}{\left[\left(C_\alpha / g_m\right)^{1/\alpha} s\right]^\alpha + 1}, \quad (10)$$

where g_m is the transconductance of both transistors (Mp1 and Mp2).

A fractional-order model that describes the dynamics of a phantom EEG measurement chain, has the the form of a high-pass filter with fixed value of the order and variable values of the time-constant and maximum gain.

It is described by the transfer function in (11)

$$H(s) = \mathrm{G}_0 \frac{(\tau s)^a}{(\tau s)^a + 1} \quad (11)$$

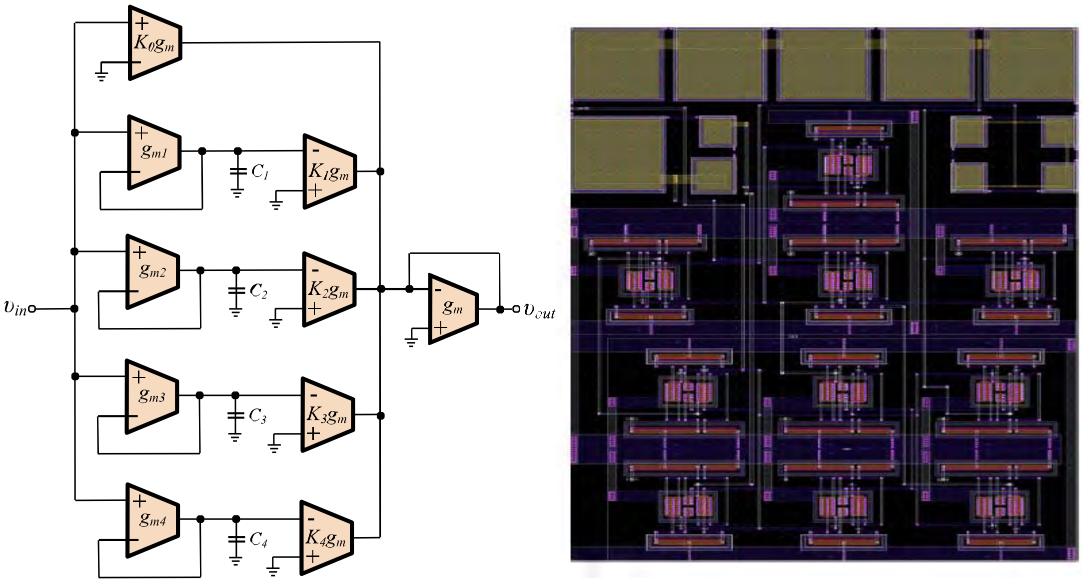

Figure 4: Fractional-order high-pass filter for phantom electroencephalographic system model implementation (a) OTA based topology, (b) layout design [16]

An OTA based implementation is depicted in Figure 4a, while the layout design is given in Figure 4b. This filter offers the benefits of the fully electronic adjustment of its frequency characteristics, minimization of the values of the required capacitances. These have been achieved by realizing the partial fraction expansion of the transfer function which approximates the Laplacian operator, instead of its direct realization [16].

Conclusion and Future Work

There are many types of filtering transfer functions and their particular implementations as shown briefly in this short task summary. Of course, choice of the particular solution is based on final application and its specific requirements. Therefore, there is still space for further research and investigation of new topologies, approaches and design methods, such as in [2-6] or [9], for instance.

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Fractional Order Derivatives in Analysis of Online Handwriting Associated with Neurodegenerative or Neurodevelopmental Disorders

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Short abstract: Graphomotor disabilities (GD), manifested in e.g. people with neurodegenerative or neurodevelopmental disorders, can have serious consequences, and can greatly affect a persons quality of life. Although the basic kinematic features such as velocity, acceleration, and jerk were proved to effectively quantify GD symptoms, a recent body of research identified that the theory of fractional calculus can be used to even improve the objective GD assessment. The aim of this chapter is to summarize our current research in this field. The online hand writing signals were parametrized by kinematic features utilizing three fractional-order derivatives (FD) approximations: Grünwald-Letnikovs, Riemann Liouvilles, and Caputos. Our results shows the differences across the employed FD approaches for the same kinematic handwriting features and their potential in GD analysis. The results suggest that the Riemann-Liouville's approximation in the field of quantitative GD analysis outperforms the other ones. Using this approach, we were able to automatically estimate the severity of the GD with 16.25% error. Based on the observed results, we suggest the Riemann Liouvilles FD approach as a most suitable candidate for a digital filter quantifying the online handwriting associated with GD.

Keywords: Fractional-order derivatives, online handwriting, neurodegenerative disorders, neurodevelopmental disorders, features extraction

Extended abstract

Handwriting is a complex perceptual-motor skill composed of a coordinated combination of fine graphomotor movements, motor planning and execution, visual perceptual abilities, orthographic coding, kinesthetic feedback, and visual-motor coordination [1]. These skills are referred to as graphomotor skills (GS) [2]. When a person suffers from a neurodevelopmental (e.g. developmental dysgraphia) or neurodegenerative (e.g. Parkinson's disease) disorder, she/he is very likely to exhibit graphomotor disabilities (GD). Such difficulties can have serious consequences, and can greatly affect a persons every-day life. To be able to introduce a timely and effective treatment and to improve a person's quality of life, neurologists, psychologists, special education counselors, and other experts need a robust framework that will enable them to diagnose GD in an objective and complex way with minimum manual intervention, cost and time constraints [3]. Nowadays, the most promising approach into computerized assessment of GD utilizes various signals describing the process of handwriting/drawing acquired by a digitizing tablet [4]. Such signals represent movement of a digitizing stylus (pen) on horizontal and vertical axis, pressure, tilt and azimuth, acquired with respect to a specific series of timestamps (referred to as online handwriting). In addition, modern digitizers have the ability to record the movement above the surface (in-air movement). As shown in a variety of studies [5, 6, 7], online handwriting provides us with the capability of going beyond the limitations of human perception and to characterize the handwriting/drawing process in terms of its kinematic, dynamic and temporal features that are not accessible from a final product using a conventional pen and paper methodology. Recently, online handwriting has been advantageously used in a variety of research studies focusing on identification and assessment of GD in children experiencing developmental dysgraphia (DD) [7], or in adults suffering from Parkinson's disease (PD), Alzheimer's disease (AD), essential tremor [5], etc, where most of these studies employed the differential derivative based handwriting features. Despite the indisputable success of these features in various use-cases, our last studies have pointed out to the necessity of additional research in the field of fractional calculus (FC) in direction of introducing a novel framework for unique and more advanced representation/parameterization techniques of handwriting/drawing. In our recent studies [4, 7, 8, 9], we established new handwriting parametrization techniques utilizing fractional-order derivatives (FD) as a substitution of the conventionally used differential derivatives in the kinematic handwriting features extraction. We identified that FD could be advantageously used to outperform some traditional approaches into GD quantification and assessment in children/adults with neurodevelopmental/neurodegenerative disorders. Since the part of our research has been motivated and supported by the COST Action CA15225, we would like to summarize the proposed usage and results of FD as a digital filter in the process of online handwriting features extraction for analysis of neurodevelopmental/neurodegenerative disorders.

Datasets

For the purpose of this work, we used two different databases. Firstly, the Parkinson's disease handwriting database (PaHaW) [10]. The database consists of several handwriting or drawing tasks acquired in 37 PD patients and 38 age- and gender-matched healthy controls (HC). The participants were enrolled at the First Department of Neurology, St. Anne's University Hospital in Brno, Czech Republic. All participants signed an informed consent form approved by the local ethics committee. Secondly we used a database that consists of 85 children (31 girls and 54 boys) attending 3rd and 4th grade at several primary schools in the Czech Republic. Children were asked to perform drawings, writings, and several cognitive tests based on a protocol consisting of 31 tasks designed in cooperation with psychologists and special educational counselors. Parents of all children participating in this study signed an informed consent form approved by the Ethical Committee of the Masaryk University. During handwriting tasks performance, the participants were rested and seated in a comfortable position. A digitizing tablet (Wacom Intuos) was overlaid with an empty paper and the participants wrote on that using the Wacom Inking pen. Online handwriting signals were recorded with f_s = 150 Hz sampling rate.

Methodology

One of the goals of our research is the investigation of several FD approximations as a new advanced approach for handwriting parameterization. We developed this method to substitute the conventional differential derivative in the feature extraction process in order to improve the quantitative analysis of the GD. In the scope of our research, we utilized three FD approximations: Grünwald-Letnikov (GL), Riemann-Liouville (RL), and Caputo (C), implemented by Valério Duarte in Matlab [11, 12]. The one with the best quantification abilities will be selected and implemented in the final digital filter. For a better understanding of the advantages of fractional-order derivatives, let us continuously recall all employed FD approaches. Firstly, the Grünwald-Letnikov approach [13]. The usual n-order derivative of $f(t)$ can be written as the limit

$$\lim_{h\to 0}\frac{\Delta_h^n[f](t)}{h^n}=\lim_{h\to 0}\frac{\sum_{k=0}^{n}(-1)^k\binom{n}{k}f(t-kh)}{h^n}, \tag{1}$$

in which $\binom{n}{k}=\frac{n!}{k!(n-k)!}$ is the binomial coefficient. Suggestively, one might generalize the difference operator $\Delta_h^n[\cdot]$ to $\Delta_h^\alpha[f]=\sum_{k=0}^{\infty}\binom{\alpha}{k}f(t-kh)$, with $\alpha > 0$. Now, the binomial coefficient is given by $\binom{\alpha}{k}=\frac{\Gamma(\alpha+1)}{\Gamma(k+1)\Gamma(\alpha-k+1)}$. Then, the Grünwald–Letnikov fractional-order derivative is given by

$$ {}^{GL}\mathrm{D}^{\alpha}[f](t)=\lim_{h\to 0}\frac{\sum_{k=0}^{\infty}\binom{\alpha}{k}f(t-kh)}{h^{\alpha}}, $$

where ${}^{GL}\mathrm{D}^{\alpha}y(t)$ denotes the Grünwald-Letnikov derivatives of order α of the function $[f](t)$, and h represents the sampling lattice.

Next, we employed FD given by Riemann-Liouville. The left-inverse interpretation of $\mathrm{D}^{\alpha}y(t)$ by Riemann-Liouville from 1869 is defined as

$$ {}^{RL}\mathrm{D}^{\alpha}y(t)=\frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dt}\right)^{n}\int_{0}^{t}(t-\tau)^{n-\alpha-1}y(t)dt, $$

where ${}^{RL}\mathrm{D}^{\alpha}y(t)$ denotes the Riemann Liouville derivative of order α of the function $y(t)$, Γ is the gamma function and $n-1<\alpha\leq n$, $n\in\mathbb{N}$, $t>0$.

Finally, we employed FD given by M. Caputo [14]. In contrast to the previous ones, the improvement hereabouts lie in the unnecessity to define the initial FD condition [15, 16]. The Caputo's definition from 1967 is

$$ {}^{C}\mathrm{D}^{\alpha}y(t)=\frac{1}{\Gamma(n-\alpha)}\int_{0}^{t}(t-\tau)^{n-\alpha-1}y^{n}(t)dt, $$

where ${}^{C}\mathrm{D}^{\alpha}y(t)$ denotes the Caputo derivative of order α of the function $y(t)$, Γ is the gamma function and $n-1<\alpha\leq n$, $n\in\mathbb{N}$, $t>0$.

Handwriting Features

Basic kinematic features from the input handwriting signals were extracted, namely velocity, acceleration, jerk and their horizontal and vertical variants. In the first step, the FD-based features were calculated for different values of α in range from 0.1 to 1.0 with the step of 0.1. Next, the most discriminative handwriting tasks were selected and deeper analyzed with a finer step of α (0.01). This selection was made in order to reduce computational cost of the analysis and to improve the performance of the FD-based digital filter. Statistical properties of all extracted handwriting features were expressed using mean, median, standard deviation (std), and maximum (max).

Statistical Analysis

At first, we performed the normality test of the handwriting features using the Shapiro-Wilk test. In the case of non-normally distributed features, we utilized the Box-Cox transformation. Next, to assess the strength of the relationship between the feature values and the clinical scales, Spearman's and Pearson's correlation coefficients were computed (we considered the level of significance equal to 0.05). During the statistical analysis, we controlled for the effect of several confounding factors. To evaluate the discriminative power of the handwriting features, we performed a multivariate classification and regression analysis. For this purpose, we employed the state-of-the-art algorithm XGBoost [17] (10-fold cross-validation with20repetitions). The XGBoost algorithm was selected, because of its ability to achieve good performance on a small data set. Classification performance was evaluated by the Matthew's correlation coefficient (MCC), classification accuracy (ACC), sensitivity (SEN), and specificity (SPE). The regression model's performance was evaluated by the mean absolute error (MAE), the mean square error (MSE), the root mean square error (RMSE), and the estimation error rate (EER).

Results

The best classification accuracy obtained by the FD-based features was ACC=87%, SEN = 82% and SPE = 90%. In comparison to the base line, we improved the classification accuracy by 10% using kinematic features only. To better understand the advantage of FD in kinematic analysis of online handwriting, we plotted vertical velocity patterns of the sentence task for different orders of FD (see Figure 1). We can observe a big difference between $\alpha = 0.1$ and the rest of the orders, including the full derivative. This large distance means that we are working with completely new information that is far from that contained in the full derivative. A comparison of an identical feature (i.e. velocity for $\alpha = 0.2$) extracted from the handwritten product associated with the GD is shown in Fig. 2. It illustrates the differences across the involved FD approximations. The velocity function extracted by the Caputo's FD dominates by significant peaks in the positions, where a subject interrupts the performance for a moment and then continues writing. These interruptions are also visible in the function computed by the Riemann-Liouville approach, though in the form of a constant line followed by elevated oscillations instead of peaks.

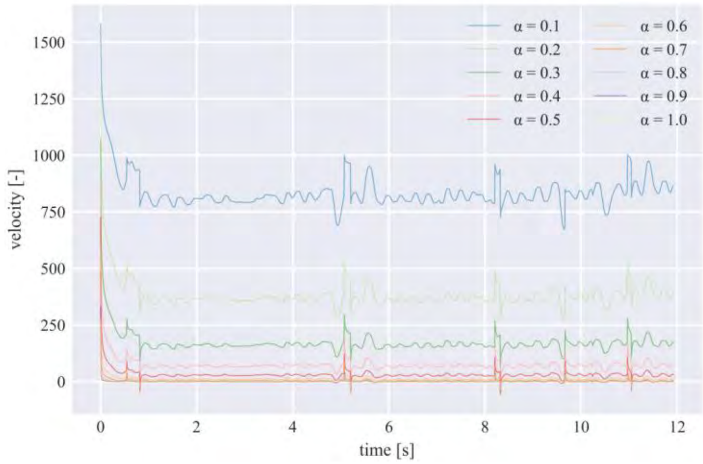

Fig. 1: Vertical velocity patterns of the sentence task for different orders of fractional-order derivatives (FD)

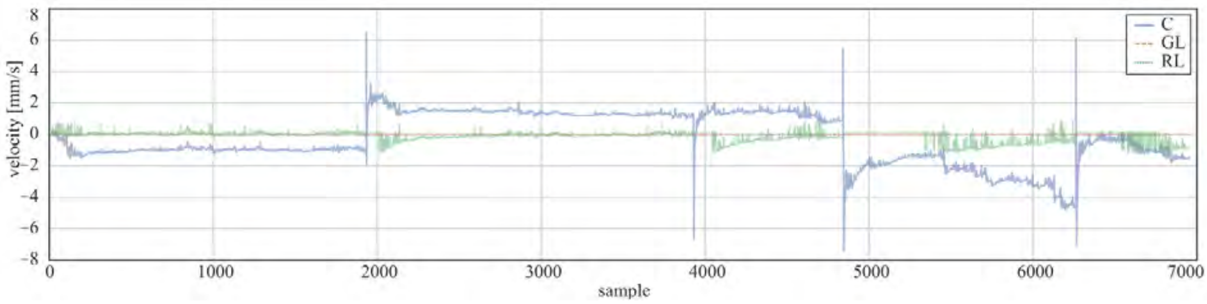

Fig. 2: Comparison of the velocity function ($\alpha = 0.2$) across all the FD approximations (a child associated with graphomotor difficulties; C–Caputo; GL–Grünwald-Letnikov; RL–Riemann Liouville).

In the scope of the correlation analysis associated with the clinical scales, the Caputos FD approach exceeds the rest of the analyzed FD approximations. However, the results of the multivariate analysis suggest that the Riemann-Liouvilles approximation in the field of the quantitative GD analysis outperforms the other ones (MAE = 0.65, i.e. error = 16.25%). Figure 3 reports the α orders of the handwriting features included in the best-performing classification and regression models of the most discriminative tasks (repetitive loops and sentence). By intersectioning optimal α ranges of classification and regression analysis, we created a final optimal range of α from 0.05 to 0.45 and from 0.60 to 0.80, that is recommended to be used in the final digital filter.

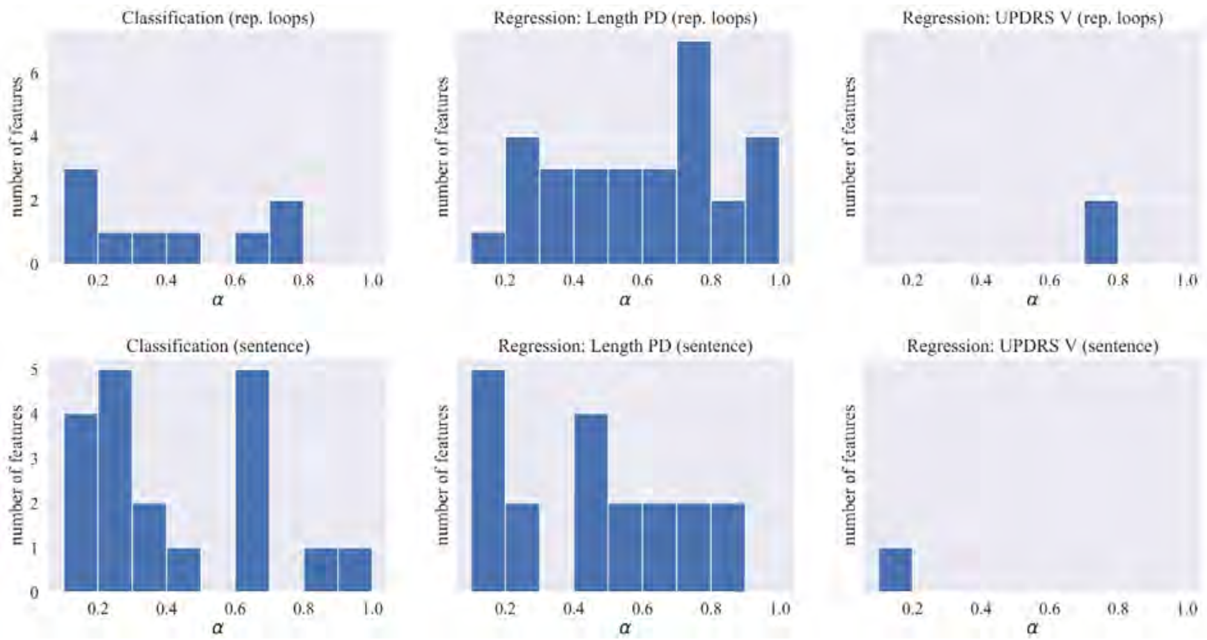

Fig. 3: Distributions of FD order α among the fine-tuned parameters

Conclusion

In this contribution, we summarized our current research in the field of GD analysis employing the FD-based kinematic features. Regarding the results, the most suitable candidate for a digital filter used during the assessment of GD is the Riemann Liouvilles FD. The identified optimal values of the FD order should be in the range from 0.05 to 0.45 or from 0.60 to 0.80. Identification of these ranges enables a significant reduction of computational cost (by approximately 50%).

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A Step Towards Practical Optimality of FO Controllers: Time-domain Identification of Fractional Order Systems for Data Driven Optimal Control

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Abstract**:** Practical performances of optimal fractional order control system designs are degraded in real-world applications because of two major factors: The first one is the limitations in mathematical modeling of the real systems. Static mathematical modeling cannot deal with the complexity, changing dynamics and environmental uncertainties such as unpredictable disturbance and measurement noise. The second one is the approximate realization of the optimal fractional order controller that may cause a loss of control benefits that are provided by the fractional order system at certain levels. To address these two points in order to maintain theoretical optimality in real-world applications, namely practical optimality, one solution might be consideration of the approximate model of fraction order controllers in the optimal controller design tasks, which can mitigate the inherent performance degradations that are caused by the convergence errors of the approximation methods (e.g. band limited approximation to frequency responses, time response mismatches etc.). The second is the use of online fractional order model identification, which is continuously updated based on the recent data from the real systems, in order to represent very recent dynamical behavior of the systems. This somewhat helps coping with model complexity and uncertainty problems that are the major limitation of mathematical modeling. When online fractional order model identification is combined together with optimal tuning rules, the practically optimal solution can come from the *data-driven fractional order control* paradigm.

Keywords**:** Model reference adaptive control, fractional-order model, fault tolerance, disturbance rejection.

Extended Abstract

To contribute to the data-driven optimal fractional order control paradigm, one firstly addressed the online time-domain identification problem of fractional order systems from the input-output measurement data from the real systems. In order to perform optimal identification of One Non-Integer Order Plus Time Delay (NOPTD-I) models, a STSM collaboration was conducted in 2018. Research outputs of this fruitful STSM collaboration was published in [2]. In this extended abstract, we present a brief review of these research papers that were written on the scope of the Task 3.3 of Cost Action CA15225 [1].

This short abstract focuses on the utilization of two fundamental numerical solution methods of fractional calculus for the identification of One Non-Integer Order Plus Time Delay with one pole (NOPTD-I) models in [2]. These are Mittag-Leffler (ML) function and Grunwald-Letnikov (GL) definition. The identification process is carried out by estimating parameters of a NOPTD-I type transfer function template according to the experimental step response data. The NOPTD-I type transfer function templates are commonly expressed in the form of

$$G_p(s) = \frac{K}{\tau s^{\alpha} + 1} e^{-Ls}, \tag{1}$$

where the parameter K is a DC gain, the parameter τ is the time constant, the parameter L is the time delay and the parameter α denotes the fractional order of this model template [3]. The differential equation form of this NOPTD-I model is written by

$$\tau D^{\alpha} y(t) + y(t) = Ku(t-L), \tag{2}$$

where the initial condition $y(0^+)$ is generally taken zero in order to obtain a transfer function model [2]. To obtain impulse responses of NOPTD-I models, two definitions of fractional order derivative are considered:

a) *Mittag-Leffler (ML) definition*:

The ML method yields a continuous time solution in infinite time series form that was written in type of ML function family [16].

$$L^{-1}\{\frac{s^{\alpha-\beta}}{s^{\alpha} \mp a}\} = t^{\beta-1}\sum_{i=0}^{\infty}\frac{(\pm at^{\alpha})^{i}}{\Gamma(\alpha i+\beta)} \tag{3}$$

When this definition is applied to equation (1) and infinite time series is truncated to p number of terms because of the finite computation time realization, one obtains an approximate solution of the impulse response without considering the time delay element [2]

$$g(t) \cong (K/\tau)t^{\alpha-1}\sum_{i=0}^{p}\frac{(-(1/\tau)t^{\alpha})^{i}}{\Gamma(\alpha i+\alpha)}. \tag{4}$$

Afterwards, to implement the time delay L for a casual system, the delay element e^{-Ls} can be implemented by using the time-shifting property of Laplace transform and setting zero to all values before the time L. Accordingly, the impulse response of $G_p(s)$ model is expressed based on the ML function as [2]

$$g_p(t) \cong \begin{cases} (K/\tau)(t-L)^{\alpha-1}\sum_{i=0}^{p}\frac{(-(1/\tau)(t-L)^{\alpha})^{i}}{\Gamma(\alpha i+\alpha)} & ,t \ge L \\ 0 & ,t < L \end{cases}. \tag{5}$$

In order to obtain a discrete time solutions for a time increment of $t = nT_s, n = 0,1,2,...$.

b) *Grunwald-Letnikov (GL) definition*:

The second approach is based on the GL definition of fractional-order derivative operator, which was defined for a non-zero sampling period T_s as [5]

$$D_t^{\alpha} y(t) \cong \frac{1}{T_s^{\alpha}}\sum_{j=0}^{p} w_j^{(\alpha)} y(t-jT_s). \tag{6}$$

$$w_0^{(\alpha)} = 1,\; w_j^{(\alpha)} = (1-\frac{\alpha+1}{j})w_{j-1}^{(\alpha)}. \tag{7}$$

Then, by using equation (6) and (7), the differential equation form of NOPTD-I model (equation (2)) is solved numerically as [2]

$$y_g(t) = \frac{T_s^{\alpha} K}{\tau w_o^{(\alpha)} + T_s^{\alpha}} u(t) - \frac{\tau}{\tau w_o^{(\alpha)} + T_s^{\alpha}} \sum_{j=1}^{p} w_j^{(\alpha)} y_g(t - T_s j) . \quad (8)$$

To obtain an impulse response of $y_g(t)$, the input $u(t)$ is taken as a discrete impulse $\delta(n)$. Then, the impulse response $y_g(t)$ for T_s time increments and a sampling of $t = nT_s, n = 0,1,2,...$, [2]

$$y_g(n) = \frac{T_s^{\alpha} K}{\tau w_o^{(\alpha)} + T_s^{\alpha}} \delta(n) - \frac{\tau}{\tau w_o^{(\alpha)} + T_s^{\alpha}} \sum_{j=1}^{p} w_j^{(\alpha)} y_g(n - j) . \quad (9)$$

To implementing time delay L for a casual system response from the model, a time shifting L is applied to $y_g(n)$, and all values before the discrete delay time $n_L = L / T_s$ was set to zero. The impulse response of $G_p(s)$ is calculated by [2]

$$g_p(n) \cong \begin{cases} y_g(n - (L/T_s)) & , n \geq (L/T_s) \\ 0 & , n < (L/T_s) \end{cases} . \quad (10)$$

The time response of $G_p(s)$ function can be found by calculating the discrete convolution form,

$$y(n) = \sum_{j=0}^{n} g_p(j) u(n - j) T_s . \quad (11)$$

Experimental Study

To solve the optimal step response fitting problem, a time domain modeling approach was implemented to identify model parameters of NOPTD-I models by using metaheuristic optimization algorithm:

$$\min E = \min \{ \frac{1}{v} \sum_{n=1}^{v} (y(n) - y_m(n))^2 \} . \quad (12)$$

A PSO algorithm was performed for the tuning of these four parameters that are represented by the particle position vectors as $X = [\alpha \ \mathrm{K} \ \tau \ \mathrm{L}]$. The time responses of NOPTD-I models are numerically calculated by using the impulse responses of NOPTD-I models.

An experimental study was conducted for the NOPTD-I model identification of the pith rotor of TRMS platform that was produced by Feedback Inc [6]. We applied step input and captured rotor angle data with 0.1 sec sampling period. By using this data, NOPTD-I model identification was carried out based on ML and GL solutions for 100 sec data and 50 sec truncated data windows. Table 1 shows NOPTD-I model parameters that were identified for the experimental data. The results are promising for practical optimality of FO controllers via contributing to realization of the data-driven optimal FO control systems.

Table 1. Parameter estimation and MSE performances of pitch rotor model identification for 100 sec and 50 sec data windows

Methods	K	τ	L	α	MSE
ML (100 sec)	0.3911	2.6314	0.0595	0.9067	0.001476
ML (50 sec)	0.360747	1.631099	1.209194	1.264333	0.001733
GL (100 sec)	0.3658	0.2080	2.0974	1.9890	0.000618
GL (50 sec)	0.358017	0.213390	1.980655	2.000783	0.001107

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Digital Fractional-Order Controllers: An Accurate and Reliable Implementation for Industrial Applications

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Abstract**:** Different incarnations of fractional-order control, most prominently of CRONE[1] – and fractional-order proportional-integral-derivative (FOPID) types, have been considered as a replacement for conventional control in the industry for a rather long time. On the other hand, the industry as a whole exhibits a certain inertia when it comes to introducing new technology which does not, at least at first glance, offer significant benefits and a considerable return on investment. Furthermore, looking at fractional-order control from a bird's eye view, one can clearly observe that one of the obstacles for industrial integration of advanced fractional-order control methods is the problem of implementation of the latter. Indeed, with conventional, widely accepted PID controllers, the implementation, at least in its most basic form, is trivial. Not so with fractional-order controllers, however, since an accurate and efficient implementation thereof is much more involved. Obviously, most controllers nowadays are digital, so in the general use case one would need to develop a digital fractional-order controller implementation that would be (1) accurate to the design specifications to ensure that the benefit stemming from the use of fractional-order control can observed (this should also convince the industrial partners that FO control has clear advantages); (2) be reliable to ensure stable operation of a given industrial plant. In this contribution, we discuss the aforementioned issues and propose a fractional-order PID controller implementation that, over the years since its conception, has been shown to offer said favorable qualities.

Extended Abstract

Proportional-integral-derivative (PID) controllers have become a true staple of industrial control [1,2]. Although the basic form of this controller is relatively simple, the feedback-based control actions are fundamental in their function: the proportional component drives the output of a given plant in proportion of and in the opposite direction to the output error signal, the integral component allows to eliminate steady-state error by accumulating the error signal, and the derivative component introduces a predictive capacity to the control loop to minimize the error due to disturbances.

Therefore, when the literature describing FOPID controllers has emerged in the 1990s pioneered by researchers such as Igor Podlubny [3], the expectation was that industrial application of FOPID controllers would soon follow. Indeed, given the two additional degrees of freedom, represented by the orders of integration and differentiation that are allowed to be real numbers instead of being fixed at unity, FOPID controllers are theoretically capable of improving the performance of a given control loop beyond the capacity of conventional PID controllers. The transfer function in the Laplace domain corresponding to the basic parallel form of a FOPID controller is

$$C(s) = K_p + K_i s^{-\lambda} + K_d s^{\mu}, \tag{1}$$

where K_p, K_i, and K_d are real numbers representing the proportional, integral, and differential gains, respectively, and $\lambda > 0$ and $\mu > 0$ are orders of the integral and differential components, respectively. It

[1] The acronym in French stands for "Commande Robuste d'Ordre Non-Entier" which means "Robust Control of Non-Integer Order" and relates to the developments of the University of Bordeaux research group based on fundamental research of Alain Oustaloup et al.

is obvious that the control structure in (1) offers more flexibility than in the case of a conventional PID controller when $\lambda = \mu = 1$.

The benefits stemming from this additional tuning flexibility have been confirmed by many studies about the application of FOPID control to industrial processes [4,5]. However, there are also many issues with industrial integration of FOPID controllers that were also outlined in [5]. One of these issues is the implementation of fractional operators which serve as the basis for the realization of the integral and differential components of the FOPID controller. The key problem is that an ideal implementation, at least theoretically, cannot be achieved using numerical methods, hence approximations must be used. One can compare that with a basic discrete-time implementation of the parallel form of a PID controller which can be summarized in a single line as

$$u(k) = K_p e(k) + K_i \sum_{i=0}^{k} e(j) + K_d (e(k) - e(k-1)),$$

where $u(\cdot)$ is the generated control law and k designates the sample number. On the contrary, for implementing a FOPID controller, one typically turns to high order conventional transfer function based approximations of the involved noninteger components. Out of those approximation methods, the CRONE method, developed by Oustaloup *et al.* [6], is among the most popular ones [7]. Many subsequent implementation attempts have either closely replicated Oustaloup's recursive method in some form [8], proposed improvements for it [9], or considered relevant modifications [10,11,12] to improve on some qualities of the approximation.

What concerns control systems, there are usually two choices when it comes to the implementation of fractional controllers: (1) the analog electrical circuit approximations (e.g., [13,14,15]), and (2) digital approximations based on discrete time analysis of the controller including, e.g., microprocessor based implementations [12,16,17]. While the analog circuit based implementations can, in general, one day lead to a true fractional-order element, when one discusses efficient industrial application, one usually considers digital implementations because hardware that can run such implementations is ubiquitous [5]. Furthermore, due to the system cost concerns of industrial community, an embedded software FO controller realization options for low-frequency process control applications such as liquid level control may be possible by using low-cost, popular microcontrollers (e.g., Raspberry Pi or Arduino [18]). For high frequency and high performance applications of FO elements, FPGA [19,20] and DSP card realizations [12] are currently available. A diagram showing this type of implementation is depicted in Fig. 1.

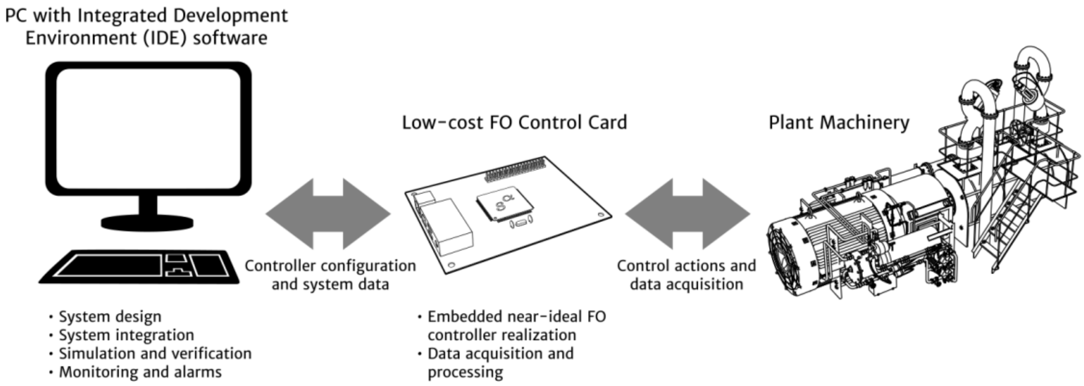

Fig. 1 A schematic diagram of an implementation of a fully integrated fractional-order control systems that include analysis, design, implementation and verification environments [5].

In this contribution, the focus is on a software based implementation of FOPID controllers mainly by combining the results from [21,22,23]. The contribution is also summarized in a conference paper [24] which marks the first public release of the relevant code to the interested parties including researchers and practitioners working in the field of control system design. It should also be noted that since we are dealing with fundamental building blocks—i.e., noninteger integration and differentiation—the same implementation methods can also be applied to other types of controllers [7,8] that make use of these fundamental blocks thus expanding the scope of the present contribution beyond just fractional-order PID controllers. Hence, the code can be reused for the purpose of creating general noninteger integration or differentiation blocks and using them as parts of other types of controllers.

Methodology

The basic methodology for implementing the digital FOPID controller corresponding to (1) has been thoroughly documented in [21], therefore in the following only key ideas are provided. The complete procedure for obtaining an approximation of the controller in the form of a digital infinite impulse response (IIR) filter can be summarized as follows:

1. Compute the approximations of the fractional integral and differential components separately using Oustaloup's method using the design parameters $[\omega_l ,\omega_h]$ (the frequency range of the approximation) and N (the order of the filter that results in a transfer function of order $2N + 1$); note that where the implementation of the FOPID is concerned, according to the discussion in [7], the integral component should be approximated such that
$$\frac{1}{s_i} = \frac{s^{1-\lambda}}{s} . \quad (2)$$
2. Convert both approximations to discrete time using the zero-pole matching equivalents method with a sampling time of t_s.
3. Instead of using the resulting transfer function directly, convert it to a second-order section representation of the form
$$H(z) = K_c \prod_{k=1}^{v} \frac{1 + b_{0k} z^{-1} + b_{1k} z^{-2}}{1 + a_{0k} z^{-1} + a_{1k} z^{-2}} , \quad (3)$$
where K_c is the gain of the transfer function and v is the resulting number of sections. This form ensures computational stability of the approximation.

This implementation can lead to practically applicable results even in case of limited computational ability of the hardware chosen for the deployment of a FOPID controller [21]. Furthermore, some recent research results confirm long-term stability of the 32-bit microcontroller based implementation of the FOPID controller [25].

It is relatively simple to compute and compare the frequency domain characteristics of both the original controller in (1) and its approximation in (3) to ensure that the obtained controller approximation leads to the correct frequency domain specifications prescribed for the control loop within the valid frequency range. Stability assessment can also be performed on this basis.

Software Implementation

The software package providing support for both Python and C/C++ is available at GitHub [26] under the MIT license.

Python implementation

The Python implementation well complements the FOMCON toolbox version for Python 3 [23] and is intended for use with Python 3. Only the FOPID realization module is implemented presently, and no support for accurate timing is provided. Python 3 is not commonly considered a language that supports hard real time computations, but can use external synchronization [24]. The availability of the Python implementation allows a wider circle of practicioners to work with FOPID controllers due to relative simplicity of using the implementation in any project.

C/C++ implementation

The C/C++ implementation is more involved. It also includes a FOPID controller tuning module. Since the code is available and reasonably well structured only the key points are recalled below.

- The initial code was used with a microcontroller based implementation discussed in detail in [21]. It was rewritten to work in a software-in-the-loop (SIL) context, but it should be relatively easy to adopt it back to the form usable in a microcontroller or embedded system.
- The arrays which hold the discrete coefficients of the second order sections in (3) are residing in a preallocated memory space to avoid potential issues stemming from dynamic memory allocation. Other variables, such as controller gains, orders, and the optimized parameters θ c along with corresponding orders are also placed in global scope.
- F-MIGO rules and creating FOPDT approximations from FO-FOPDT are not yet implemented. The control designer is responsible for setting λ and μ manually.

The current example implementation also uses the socket approach as does the Python implementation. With respect to this, the code should work without modification in Windows environment. For Linux, the socket communication part must be rewritten.

Having both a Python and a C/C++ implementation of a FOPID controller readily available to researchers and practicioners ensures sufficient coverage of the target groups that are involved in automatic control design. Recent research results confirm the accuracy and reliability of the proposed controller implementation [21 , 22 , 25]. On the other hand, the technology readiness level is still modest. It is expected, especially due to the availability of the Python implementation, that deployment of FOPID controllers will be facilitated, however, thus leading to wider industrial adoption of the latest research results.

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Automotive applications

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Abstract: This abstract illustrates recent results in fractional modeling and control of complex automotive systems. More specifically, the activities in the COST Action CA15225 focused on innovative injection systems reducing pollution and consumption in both Diesel and CNG internal combustion engines. The contributors proved that fractional-order modeling and control, as opposed to integer-order modeling and control, may offer benefits that are sometimes unpredicted or underestimated by researchers and practitioners.

Keywords: Fractional calculus; fractional-order modeling; fractional-order control; fractional-order PI controller; Diesel engine; CNG engine; electro-injector; automotive applications.

Extended Abstract

In the past years, ever growing efforts were devoted to new strategies for reducing fuel and energy consumption as well as polluting emissions of automotive engines. A continuous improvement of technologies was finalized to limit fuel consumption and operating costs, reduce harmful emissions (CO2, CO, NOx, HC, particulate matter, noise, etc.) and adhere to increasingly strict regulations. A goal in EU for 2030 is reducing greenhouse gases (GHGs) emission by at least 40% below 1990 levels, thanks to the increase of renewable energy, and improving energy efficiency by 27%. The goal for 2050 is reducing GHGs emission by 80%-90% and GHGs emission from transport by 60%. In common rail injection systems (CRIS), the advances in technology and combustion processes allowed a more precise metering of the air/fuel mixture demanded by the variations of engine speed and load, then a better performance and lower harmful emissions in every working condition. However, continuous developments are required to improve the metering that poses severe specifications on the fuel injection pressure, which can be controlled with accuracy, and on the injection timing. One main issue is then to develop accurate models for representing and controlling the injectors. However, fuel dynamics is complex and experimental tuning is hard, making the common rail injection control difficult and highly imprecise. Hence, fractional order models can significantly improve the prediction capabilities or reduce the model complexity, whereas advanced fractional-order controllers may guarantee better performance/robustness than integer-order counterparts. Here, two different applications are considered employing the CRIS technology. The first considers an electro-injector for Diesel engines. The second is based on compressed natural gas (CNG). The aim is showing the benefits, which are sometimes unpredicted or underestimated, by fractional-order models and controls. The above considerations motivated the activities in Task 4.1 of WG4, partially developed in cooperation with Professors Milan Rapaic, Roberto Garrappa, and Sverre Holm. The work partly relies on the outputs of Task 1.3, by extending the research results on parameter estimation of innovative engines, and on the outputs of Task 2.3 [1].

Fractional-Order Modeling Fuel Flow in an Electro-Injector of a CR Diesel Injection System

To achieve an efficient combustion, the electronic control unit (ECU) of a Diesel CRIS must accurately meter the amount of fuel and the air-fuel mixture that is injected into the cylinders, even by implementing injection rate shaping (IRS) strategies to obtain a desired profile of the flow rate. As for the electro-injectors, an accurate model is beneficial for optimizing their layout, parameters, and operation and for controlling IRS. The section view of the considered CR electro-injector and its main parts is shown in Fig. 1.

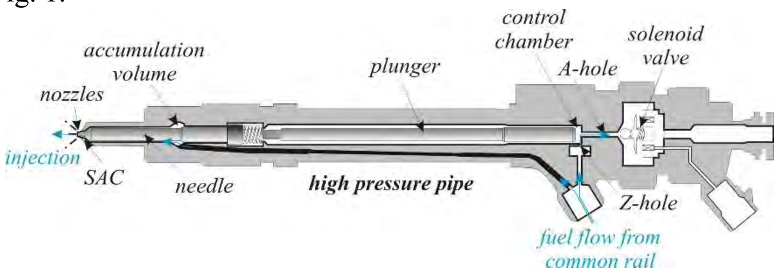

Fig 1. Electro-injector in a common rail Diesel injection system, with its characteristic pipe to deliver fuel from the common rail volume

Basically, two circuits can be distinguished: a control circuit and a feeding circuit. In the control circuit, fuel arriving from the common rail passes through orifices and enters a "control chamber", in which the fuel pressure determines if the plunger-needle element is pushed down or up. In the feeding circuit, fuel arriving from the common rail flows through a high-pressure pipe, enters an accumulation volume and then a terminal "SAC" volume, from which it is injected into the cylinders when the nozzles are open. Fuel flow is regulated by a solenoid valve, which is employed to change the pressure acting on the plunger in the control chamber.

The fuel is compressible, the feeding pipe is subject to elastic deformation, and distributed friction losses occur. Cavitation through the holes are considered. A complete mathematical model of the electro-injector is obtained by partitioning it in connected volumes, in which fuel is accumulated. For some of these volumes, a lumped parameters representation by ordinary differential equations is suitable because the pressure is uniform and time-varying and there is no wave propagation. For the high-pressure pipe between the common rail and the accumulation volume, a distributed parameters representation is necessary to describe the wave pressure propagation. To this aim, the classical Navier-Stokes partial differential equations can be suitably extended by introducing fractional-order derivatives to improve model accuracy, resulting in the modified momentum and continuity equations:

$$\frac{\partial^\alpha p}{\partial t^\alpha} + c_0^2\,\rho_0^2\,\frac{\partial u}{\partial x} = 0 \tag{1}$$

$$\frac{\partial^\beta u}{\partial t^\beta} + \frac{1}{\rho_0}\,\frac{\partial p}{\partial x} = \nu\,\frac{\partial^2 u}{\partial x^2} = 0 \tag{2}$$

where $p = p(t, x)$ is the fuel pressure depending on time t and on the location x on the (unique) direction of propagation along the pipe, $u = u(t, x)$ is the fuel wave velocity, $c_0 = c_0(p)$ is the speed of sound, $\rho_0 = \rho_0(p)$ is the density, $\nu = \nu(p)$ is the kinematic viscosity, and the non-integer orders are $0 < \alpha \leq 1$ and $0 < \beta \leq 1$. To improve the model prediction, non-integer orders and the values of mechanical parameters have been optimized. An evolutionary technique based on Differential Evolution is employed to overcome the nonlinearity and complexity of the problem. Equations (1)-(2) can not be analytically solved. Then, a numerical method is employed by using boundary conditions, i.e. the instantaneous pressures and flows at the pipe inlet and outlet sections. The method is based on finite differences. Discretization of time-fractional derivatives uses the Grünwald-Letnikov (GL) scheme:

$$\frac{\partial^\alpha}{\partial t^\alpha} p(t_n, x) \approx \frac{1}{\tau^\alpha}\left[p(t_n, x) + \sum_{j=0}^{n-1} \omega_{n-j}^{(\alpha)} p(t_j, x)\right] \tag{3}$$

$$\frac{\partial^\beta}{\partial t^\beta} u(t_n, x) \approx \frac{1}{\tau^\beta}\left[u(t_n, x) + \sum_{j=0}^{n-1} \omega_{n-j}^{(\beta)} u(t_j, x)\right] \tag{4}$$

where the coefficients of the GL scheme are recursively evaluated as $\omega_j^{(\alpha)} = \left(1 - \frac{\alpha+1}{j}\right)\omega_{j-1}^{(\alpha)}$ and

$\omega_j^{(\beta)} = \left(1 - \frac{\beta+1}{j}\right)\omega_{j-1}^{(\beta)}$. The approximations are inserted in the momentum and continuity equations in a mixed implicit/explicit way. Discretization of space derivatives is based on partitioning the pipe in several distinct volume cells. A staggered grid is obtained (Figure 2). Pressure is computed at the center of each cell, and speed is computed on every face between two adjacent cell faces, $x_{k+\frac{1}{2}} = \frac{(x_k + x_{k+1})}{2}$. Then $P_k(t) = p(t, x_k)$ and $U_{k+\frac{1}{2}}(t) = u\left(t, x_{k+\frac{1}{2}}\right)$.

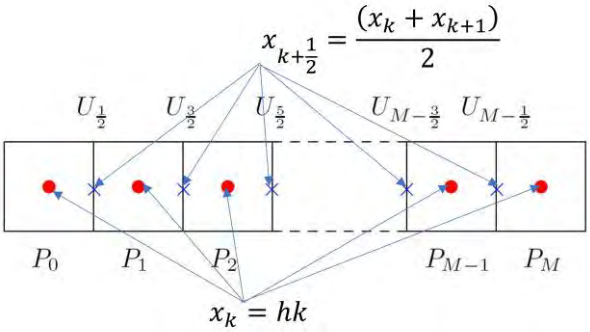

Fig. 2. Staggered grid for spatial discretization of pressure *P* (ball symbol) and velocity *U* (cross symbol)

The fully discretized version of the model consists in the following two linear systems of algebraic equations:

$$\begin{cases} (I - L_2 A_{11})\bar{P}^n = -\sum_{j=0}^{n-1} \omega_{n-j}^{(\alpha)} \bar{P}^j - L_3 A_{12} \bar{F}^{n-1} + L_2 \hat{P}^n \\ \left(I - \frac{\tau^\beta R}{hc^2} \tilde{A}_{22}\right) \bar{U}^n = \sum_{j=0}^{n-1} \omega_{n-j}^{(\beta)} \bar{U}^j - L_1 A_{21} \bar{P}^n - L_1 \tilde{P}^n \end{cases} \tag{5}$$

with $\bar{P}^n = (P_1^n, P_2^n, \cdots, F_{M-1}^n)^T \in \mathbb{R}^{M-1}$, $\bar{U}^n = \left(U_{\frac{1}{2}}^n, U_{\frac{3}{2}}^n, \cdots, U_{M-\frac{1}{2}}^n\right)^T \in \mathbb{R}^M$,
$\bar{F}^n = \left(F_{\frac{1}{2}}^n, F_{\frac{3}{2}}^n, \cdots, F_{M-\frac{1}{2}}^n\right) \in \mathbb{R}^M$, $\hat{P}^n = \left(P_{CR}(t_n), 0, \cdots, 0, P_{AV}(t_n)\right)^T \in \mathbb{R}^{M-1}$,
$\tilde{P}^n = \left(-P_{CR}(t_n), 0, \cdots, 0, P_{AV}(t_n)\right)^T \in \mathbb{R}^M$, $L_1 = \frac{\tau^\beta}{\rho h}$, $L_2 = \frac{c^2 \tau^{\alpha+\beta}}{h^2}$, $L_3 = \frac{c^2 \rho \tau^{\alpha+\beta}}{h}$, and A_{11}, A_{12}, A_{21}, $\tilde{A}_{22}$ block diagonal matrices.

To assess the model performance, simulation results obtained in the MATLAB/Simulink environment (Figure 3) were compared with experimental data measured on a real injector. Model parameters were derived from available geometric data and from experimental tests carried out in different operating conditions. Experimental data for the injected flow rate are available in the following conditions: the exciting time interval of injectors is 700 µs, the common rail reference pressure is 800, 1200, and 1600 bar. Then simulation compared results with these data (Figures 4-6). Moreover, the real common rail pressure is used as input to the pipe. A constant uniform pipe section is assumed, the time discretization points are $3000 \leq N \leq 10000$, the space discretization points are $20 \leq M \leq 100$. The non-integer orders α and β were varied to evaluate the effect on model prediction capability.

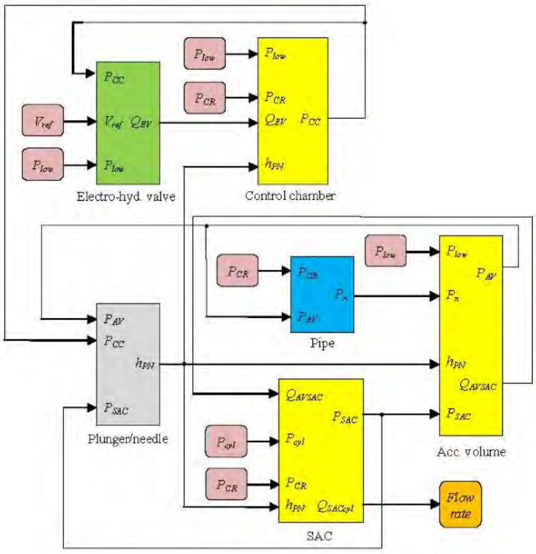

Fig. 3. Simulink block diagram

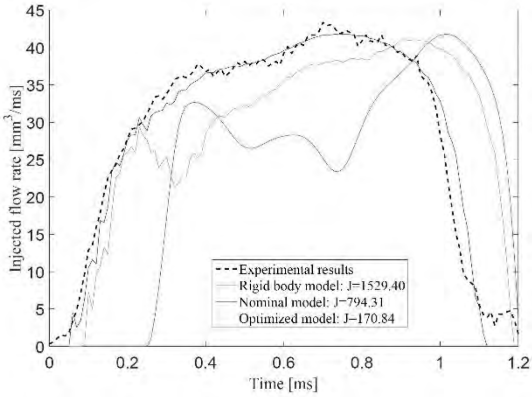

Fig. 5. Flowrate prediction at 1200 bar

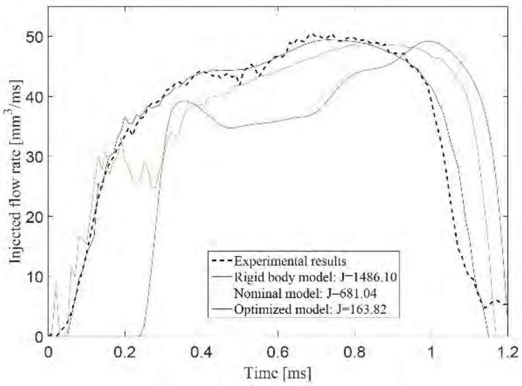

Fig. 4. Flowrate prediction at 1600 bar

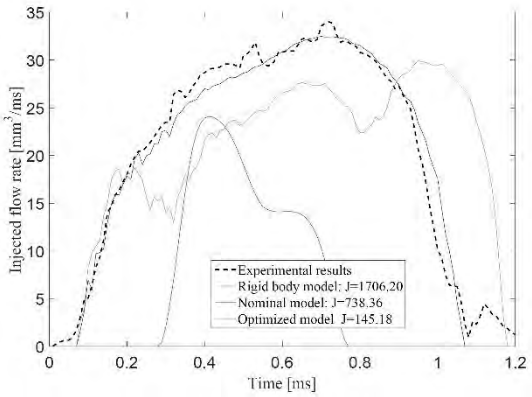

Fig. 6. Flowrate prediction at 800 bar

Fractional-Order Modelling of Common Rail Pressure in CNG Engines

The second application involves the CNG injection systems, which are considered because of low cost, availability in many countries, and the obvious capability to reduce pollution from harmful gaseous emissions if compared with Diesel or gasoline engines. In this case, obtaining the desired levels of the injection pressure is a challenge, and defining the most adequate control technique is an open problem. Namely, the gas compressibility makes the working point of the CNG injection system change a lot, on dependence of the speed and power requested by the driver, so that complex phenomena affect the performance. A possible solution is to improve the robustness and performance of the controllers by advanced schemes and by model-based design approaches. Here, it is shown how to represent the pressure dynamics in the injection system by identifying a linear non-integer-order model that describes the process in a more effective and compact way than standard integer-order models of high order.
The system under study is represented in Figure 7. The gas is delivered from a high-pressure tank (40-200 bar) to a common rail volume (4-20 bar). Gas flows through different pipes and passes through a mechanical pressure reducer, in which a piston separates a control chamber from a main chamber, in its turn linked to the common rail volume. A solenoid valve regulates the flow into the control chamber. The valve and all the process are controlled by an ECU, which determines the common rail pressure and controls gas flow by setting the injection timing and the current driving the valve.

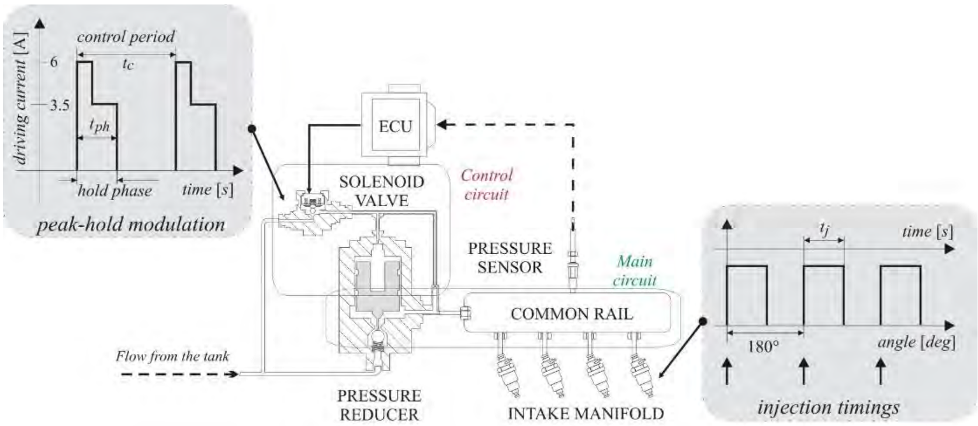

Fig. 7. Operation of the CNG injection system.

The model of the injection system must trade-off between accuracy and simplicity and consider the suitability for control development. Complex behaviors incorporating both temporal and spatial dynamics can be suitably described by PDEs. However, when the spatial component of the process dynamics is not of interest, and only input-output relationships are considered, these processes are described by various forms of irrational transfer functions and, in many cases, the resulting behavior is best modeled by sets of fractional differential equations. Besides, a simplified fractional order model would be more appropriate for control purposes than a distributed-parameters model, and could be easily derived by an identification process based on real experimental data. The two-steps approach presented here enables the derivation of a linear fractional-order model of the CNG injection system. The first step aims at fitting time-domain data by using an intermediate ARX model of high degree, whose parameters are estimated using the least-squares method. The model is over-parameterized so that all the process dynamics is captured. The evaluation of its frequency response allows to decide which class of linear models is the most suitable one for the process under consideration. The second step involves the derivation of the linear fractional-order model from the identified ARX model. As shown in figure 8, the amplitude characteristic of the ARX model is approximately piece-wise linear, with two critical frequencies and slopes which are not integer multiples of 20 dB/dec. Then, it is possible to obtain a fractional model of simpler structure, with less parameters, yet similar descriptive capabilities. Therefore, a model of the following structure can be postulated:

$$G(s) = k \frac{\left(\frac{s}{\omega_2}+1\right)^{\alpha_2}}{\left(\frac{s}{\omega_1}+1\right)^{\alpha_1}\left(\frac{s}{\omega_3}+1\right)^{\alpha_3}} \tag{6}$$

Unknown model parameters are the critical frequencies and orders. To obtain the parameter values, a Particle Swarm Optimization algorithm considered the following cost function,

$$J = \sum_{i=1}^{N} \left| 20log|G(j\xi_k)| - 20log\left|\frac{B(e^{j\xi_k T})}{A(e^{j\xi_k T})}\right| \right| \tag{7}$$

which considers the amplitude response discrepancy between the fitted ARX model and the fractional model which is being identified. ξ_k are adjacent angular frequencies chosen to be uniformly logarithmic distributed in the range from $\omega_{min} = 0.1$ rad/s to $\omega_{max} = 100$ rad/s. $N = 100$ was used.

Fig. 8 shows the agreement of the ARX model with the fractional-order one in the frequency domain. Fig. 9 compares the fractional-order model output and the experimental results in the time domain. Validation of the fractional-order model is assessed by considering different operating conditions: a constant 40 bar tank pressure, a 2400 rpm engine speed, a 8 ms injectors exciting time interval, and successive step variations of the solenoid valve control signal. As expected, the results show that the fractional-order model well approximates the real system behavior both during the rising and drop transients. The model is also able to catch the oscillating behavior within the control period, with pressure variations of the same amplitude.

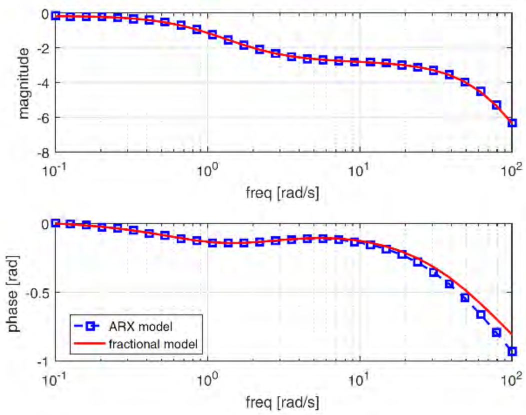

Fig. 8. Relevant part of the frequency characteristic of the optimal ARX model, and the optimal fractional-order model.

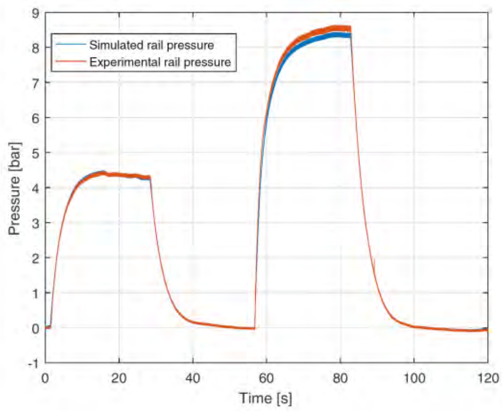

Fig. 9. Comparison between experiments and simulations (fractional-order model) for successive step variations of the solenoid valve driving signal.

Fractional-Order Control of Common Rail Pressure in CNG Engines

Parametric variations due to substantial changes of the injection system working points, complex fluid-dynamic phenomena, and disturbances, can degrade the pressure control performance in CNG injection systems. For this reason, the common integer-order PID controllers are not the best solution for the injection pressure control. Conversely, Fractional-order PI (FOPI) controllers improve robustness for each considered working point. Here, it is presented a control scheme which combines a systematic design methodology of FOPIs and a gain scheduling technique. The design method is based on a linearized model of the CNG injection system, which is derived by applying physical laws (continuity law, conservation of momentum, Newton's second law). The model inputs are the command to the valve (u_1) and to the injectors (u_2), the last being considered as a disturbance, while the output is the rail pressure: $y = x_2$. The pressure dynamics can be represented by a first-order transfer function:

$$G_p(s) = \frac{K_e}{1+T_e} e^{-L_e s} \tag{8}$$

where K_e is the equivalent static gain, T_e is the equivalent time constant, L_e is the equivalent dead-time. The injection system works in several conditions that depend on the driver power request, the engine speed, and the applied load. It follows that the triple (K_e, T_e, L_e) depends on the working point, in particular on the pressure in the tank, resulting in a family of models. However, L_e can be assumed constant to represent the pressure propagation delay from the main chamber to the common rail.
The proposed control methodology employs FOPI controllers and gain scheduling. The controllers are expressed by the following transfer function:

$$G_c(s) = K_P + \frac{K_I}{s^{\nu}} = \frac{K_I\,(1+T_I\, s^{\nu})}{s^{\nu}} \tag{9}$$

Gain scheduling is used to switch between FOPI controllers that are designed for different working points to cope with system nonlinearities. In more details, the controllers are designed for specific reference working points by a loop-shaping technique, which is reinforced by the *D*-decomposition methodology, a classical approach for robust stability analysis. The loop-shaping technique allows to achieve a good tradeoff between frequency-domain performance specifications for an optimal feedback system and robustness specifications for a nearly constant phase margin in a sufficiently wide frequency range.
The controller design takes advantage of the Bode's idea on the optimal open-loop frequency response [2]: it consists in shaping the asymptotic gain diagram, mainly in choosing the slope of the segment crossing the frequency axis, and maintaining this slope in a wide frequency interval around the crossover frequency. Hence, the phase will have a nearly flat trend and the phase margin will be constant in the

same interval. This characteristic is a clear indication of stability robustness even for high gain variations. Moreover, to obtain an optimal feedback system in the Kalman's sense, in a unitary feedback loop with the closed-loop transfer function $F(s) = 1/[1 + G^{-1}(s)]$, a high open-loop gain $|G(j\omega)|$ would be required for each ω, such that it holds $|1 + G^{-1}(j\omega)|\approx 1$ and $|F(j\omega)|\approx 1$. Namely, this condition would imply an almost perfect input-output tracking. To avoid stability problems, $|G(j\omega)|$ is shaped to get high gains at low frequencies and a roll off at high frequencies. The practical procedure is in two steps: *a*) choose the bandwidth in which optimality is desired and determine the crossover frequency where to guarantee a specified phase margin; *b*) determine the fractional integrator so that the phase plot of the open-loop gain is nearly flat (constant phase margin PM_S) in a sufficiently large range around the crossover frequency. It can be demonstrated that these conditions are fulfilled by setting the controller parameters as follows:

$$T_I = \frac{\omega_C T_e + tan(\omega_C L_e)}{\omega_C{}^{\nu}[(S - \omega_C T_e C) - (C + \omega_C T_e S)\, tan(\omega_C L_e)]} \tag{10}$$

$$\nu = 2 - \frac{PM_S}{\pi/2} \tag{11}$$

$$K_I = \frac{\omega_C^{\nu}}{K_e}\sqrt{\frac{1 + \omega_C^{\nu} T_e^2}{1 + 2T_I\omega_C^{\nu} C + T_I^2\omega_C^{2\nu}}} \tag{12}$$

The loop-shaping design technique guarantees a robust control system. The *D*-decomposition methodology allows to determine the entire set of controller gains leading to a stable closed-loop system. If PI/PID controllers are used, this set is defined in a two- or three-dimensional space and, once the gains are fixed, a point in the set is determined. The gain scheduling to switch between FOPI controllers is based on a sensitivity analysis of model coefficients.

Simulation tests were performed by using a nonlinear accurate model of the CNG injection system, which was implemented by the AMESim virtual prototyping tool. During the simulations, typical value for the reference pressure p_{CR} and injection timing t_{inj} were considered, each yielding a different triple (K_e, T_e, L_e), then a different controller. The aim was to compare PI and FOPI controllers, both gain scheduled in the same way. The most important result should be limiting the overshoot in the actual common rail pressure. Namely, overshoot would imply too much injected fuel, which alters the air-fuel ratio and increases consumption and emissions. The simulation experiments considered step variations of reference pressures, so that the gain scheduling determined the switch between three FOPI or PI controllers. As shown in Figure 10, the FOPI yielded better and smoother responses with reduced overshoots, and nonlinearities considerably affect performance of PI controllers.

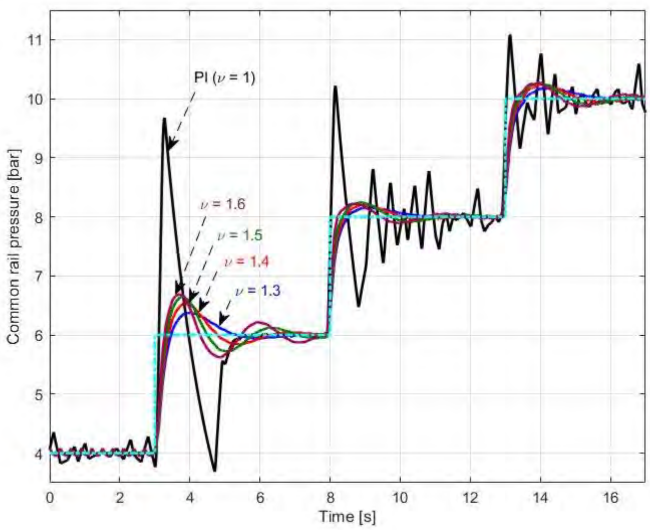

Fig. 10. Rail pressure in CNG injection system in response to a large step.

Conclusions

The common rail pressure control is a task made difficult and highly imprecise due to nonlinearities and complex phenomena involved in the injection process. However, this work demonstrates that the adoption of control-oriented models including fractional dynamics or non-integer order characteristics can significantly improve the prediction capabilities or reduce the model complexity, whereas advanced control schemes taking advantage of fractional-order controllers guarantee better performances than the standard integer-order controllers.

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A Multi-loop Control Structure for Model Reference Adaptive Control of Fractional-order PID Control Systems

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Abstract: Adaptive control is a central topic for robust performance of practical control systems because of uncertainties in real-world applications. These real-world uncertainties are considered as unpredictable disturbances, model perturbations, alteration operating conditions, effects of aging etc. in control system design. Model reference adaptive control (MRAC) is one of the successful approaches in the adaptive control topic, and therefore the possible contribution of fractional calculus to MRAC should be researched in Task 4.3. In this abstract, we present a brief review of our research papers that were written on the scope of WG4 of Cost Action CA15225 [1]. Specifically, a methodology for the utilization of fractional-order system modeling in MRAC was shown, and possible advantages of the presented method for fault-tolerance and disturbance rejection were discussed.

Keywords: Model reference adaptive control, fractional-order model, fault tolerance, disturbance rejection.

Extended Abstract

Real world performances of model-based control systems strongly depend on accurate modeling of the controlled systems. Researchers commonly agreed on the conclusion that a key contribution of fractional calculus to engineering and science problems is the exploitation of the modeling potentials in the fractional calculus for the system modeling [2]. Fractional-order system models can provide more accurate representation of real-world phenomenon. It is an expectable result because the integer-order dynamic system modeling is indeed a subset of fractional-order dynamic system modeling. Therefore, potentials of fractional calculus have manifested itself in infinitely expanding the modeling spaces of dynamical systems towards the space of non-integer-order differential equations. This significantly enhances the frequency domain characterization performance of the dynamic system models by allowing fractionally adjustment of amplitude and phase responses of the systems. Control engineering benefited from this property and the research efforts have widely focused on migration of the classical control structures towards fractional-order control domain. The fractional-order PID (FOPID) controller family was investigated extensively and the related issues such as stability, optimal tuning were addressed in many works [3-7]. Previously, the fractional-order MIT rule was suggested by Vinagre et al and improvement of fractional-order derivative operator on the MIT rule was discussed. However, this study did not directly address the utilization of fractional-order modeling in MRAC and an efficient, straightforward integration of FOPID control loop and MRAC loop. This hierarchical integration strategy in a multi-loop architecture can achieve a control performance of FOPID control system that is combined with adaptation skills of MRAC [8]. We also anticipated that the fractional-order modeling can also contribute to the MRAC structure by providing more relevant reference models. These points are the motivations of our studies related to Task 4.3 of WG4 and Task 3.3 of WG3 in Cost Action CA15225. With collaboration of Prof. Eduard Petlenkov and Dr. Aleksei Tepljakov from Tallinn University of Technology, we investigated the utilization of fractional-order modeling in conventional MRAC structure and demonstrated some significant advantages of incorporation of optimal FOPID control system and MRAC structure for the fault-tolerance and disturbance rejection [9,10].

A multi-loop MRAC-FOPID Control: Incorporation of Model Reference Control and FOPID Control Loops

Figure 1 shows an incorporation of MRAC and FOPID control loops: The inner loop is the FOPID control loop that provides stability and optimal control of the system, and the outer loop is a MRAC structure that increases the adaptation skill of FOPID loop for improvement of robust control performance for disturbances, faults or model perturbations. We observed that when the reference model of MRAC is chosen as a fractional-order model of the inner closed loop FOPID control loop, such a hierarchical, multi-loop MRAC-FOPID control structures inherently exhibit an enhanced robust control performance compared to the conventional FOPID control systems [9,10]. The adaptation skill, which is provided by MRAC loop, can increase disturbance rejection [10] and fault tolerance [9] performances of FOPID control systems. Stability issues of the stand alone MRAC can be resolved by the FOPID control loop. This mutual interaction between both loops can overcome the shortcoming of the each loop while operating alone. Figure 2 depicts these mutual benefit interactions between loops.

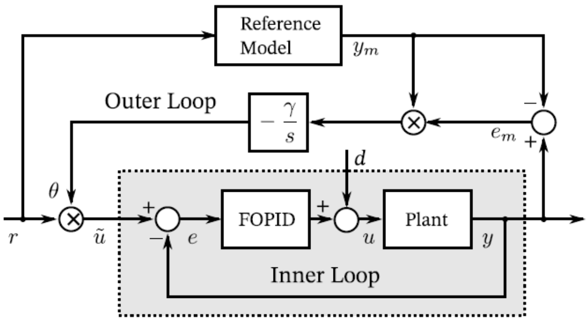

Fig. 1. Block diagram of the multi-loop MRAC-FOPID control structure [9]

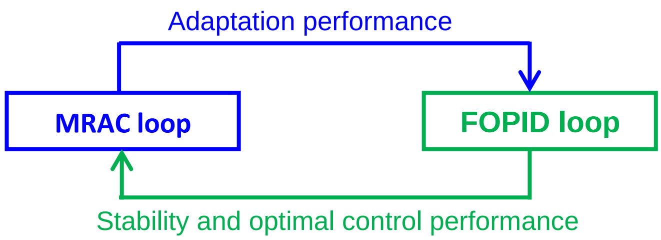

Fig. 2. Interaction schematic that illustrates integration benefits of MRAC and classical FOPID control loops

Design tasks of the proposed multi-loop MRAC-FOPID system can be summarized as [8,9]:

(i) Optimal design of the closed loop FOPID control loop: The designer optimally tunes FOPID controller in the inner loop according to one of the closed loop FOPID tuning methods [3-7].

(ii) Determination of reference model: The designer applies the closed loop model identification to FOPID control loop and obtains a fractional-order transfer function model of the closed loop control system when it is in well-tuned, fault-free and optimal state.

(iii) Appending MIT rule to the closed loop control system: After identification of the reference model, the outer loop performing MIT rule is connected to the reference input (r) of the closed loop system by means of a multiplier block as shown in Figure 1. This makes closed loop FOPID control system an inner control loop of the overall multi-loop system.

The one major advantages of this configuration appears in this design process. By ignoring item (i), one can apply it existing FOPID control system without configuring or changing any parameter of the FOPID controller. It allows easily upgrading of the operating FOPID control loops to MRAC-FOPID control system.

A Theoretical Background of Multi-loop MRAC-FOPID Control Loops

Let's assume a closed loop FOPID system that is well-tuned and in a fault-free condition. Fractional-order transfer function model of this system can be expressed in a general form as

$$G_0(s) = \frac{\sum_{i=0}^{m} b_i s^{\varphi_i}}{\sum_{i=0}^{n} a_i s^{\alpha_i}} . \tag{1}$$

The transfer function of FOPID controller is commonly written by

$$C_0(s) = k_p + k_i s^{-\lambda} + k_s s^{\mu} . \tag{2}$$

In this case, the reference model of MRAC loop (outer loop) can be expressed as [9,10],

$$T_m(s) = \frac{C_0(s)G_0(s)}{1 + C_0(s)G_0(s)} , \tag{3}$$

After determination of the reference model, MRAC loop, which performs well-known MIT rule for adaptation, is connected to inner loop (closed loop FOPID control loop) by applying input-shaping as shown in Figure 1. After connecting the outer loop to the inner loop, the input of closed loop system becomes $\tilde{u} = \theta r$, where the adaptation gain θ is determined according to the MIT rule that performs a gradient descent optimization in order to minimize the cost function, given by,

$$J = \frac{1}{2} e_m^2 . \tag{4}$$

The MIT rule for MRAC design was expressed for this system as [9,10]

$$\frac{d\theta}{dt} = -\gamma \frac{dJ}{d\theta} = -\gamma e_m \frac{de_m}{d\theta} . \tag{5}$$

Then, by considering Figure 1, one can write the model error in the form of

$$e_m = y - y_m = T(s)\theta r - T_m(s)r . \tag{6}$$

Here, the sensitivity derivative of the system is found as

$$\frac{de_m}{d\theta} = T(s)r, \tag{7}$$

where $T(s)$ represents the current transfer function of the closed loop FOPID control system (inner loop) in operation. When the reference input is substituted with $r = y_m / T_m(s)$ in equation (7) and the sensitivity derivative can be rearranged as

$$\frac{de_m}{d\theta} = \frac{T(s)}{T_m(s)} y_m . \tag{8}$$

By using it in equation (5), the MIT rule for the update of adaptation gain is written by

$$\theta = -\gamma \frac{1}{s} \left(\frac{T(s)}{T_m(s)} y_m e_m \right) . \tag{9}$$

Numerical and Experimental Results

We tested the multi-loop MRAC-FOPID structure in experimental Magnetic Levitation (ML) system. ML system introduces a highly nonlinear control problem. Figure 3 shows a picture of experimental ML system that was developed by INTECO. Experiments were conducted in Center for Intelligent Systems, Tallinn University of Technology.

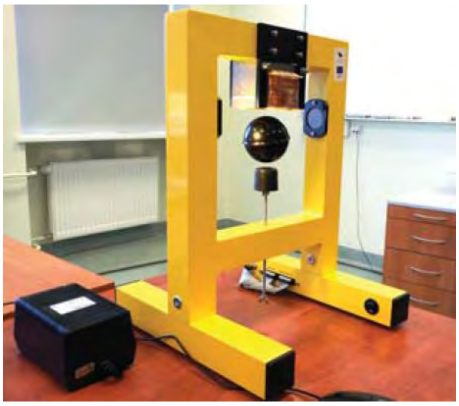

Fig 3. A picture of ML experimental system [10].

Figure 4 shows numerical results that were obtained from simulation of this experimental system in MATLAB/Simulink environment. The figure compares response of the MRAC-FOPID structure and response of the conventional FOPID structure in ML control problem. The set-point of ML system is configured 0.012 m. After settling to level of 0.012, a step disturbance was applied at simulation time 200 sec. We observed that the multi-loop MRAC-FOPID control structure can considerably improve the disturbance rejection control performance when compared to the conventional FOPID control system.

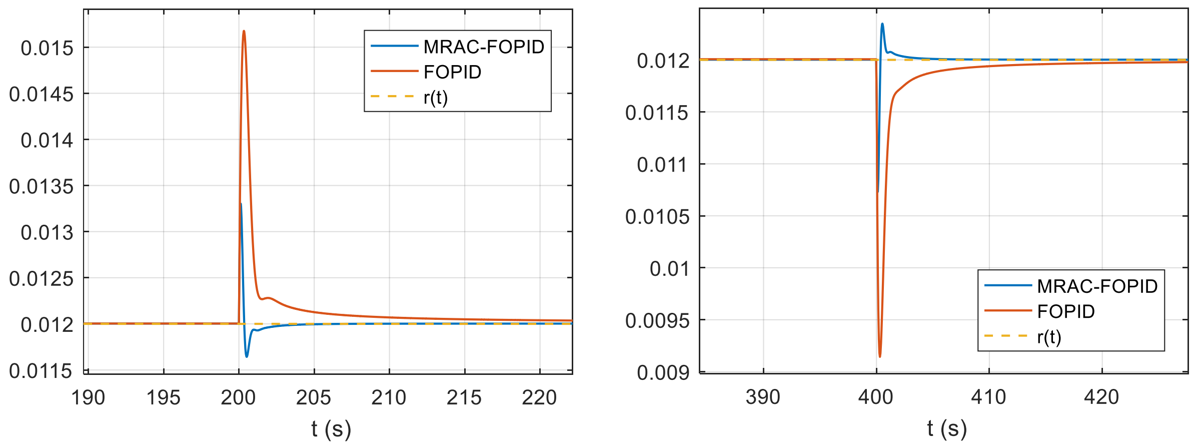

Fig 4. Simulation results that demonstrates disturbance rejection of the MRAC-FOPID structure and response of the conventional FOPID structure in ML control problem [10]

Figure 5 shows the experimental results obtained from experimental ML control system. In this system, the MRAC loop (outer loop) was first disabled and ML system output (the level of ball), control law (controller actions) were measured by applying square waveform input disturbance signals. Then, MRAC loop (outer loop) was enabled and the same kinds of measurements under the same condition were carried out. We observed that multi-loop MRAC-FOPID (MRAC enabled) can enhance control performance.

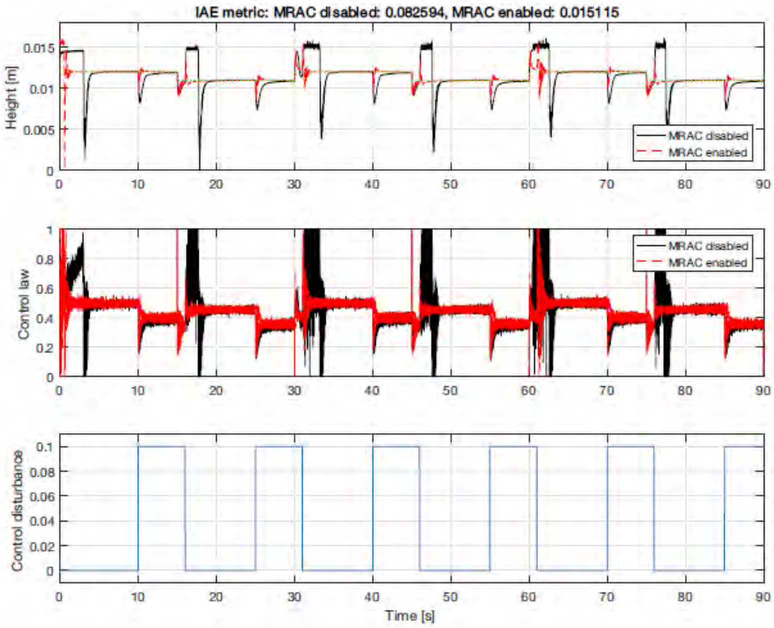

Fig 5. Experimental results obtained from control of experimental ML control system by using the multi-loop MRAC-FOPID (MRAC enabled) and the conventional FOPID control (MRAC disabled) [10]

Conclusions

Our experimental and theoretical finding indicates that multi-loop MRAC-FOPID control structure can improve the robust control performance of the conventional FOPID control loops. For practical point of view, this structure also presents advantages of an easy adaptation to existing control loops via a closed loop model identification task.

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Modelling of capturing pain pathways during anesthesia

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Introduction

After capturing the attention of both healthcare professionals and research groups, misuses of chronic pain and pain in general have witnessed a huge leap forward in our understanding. This is due to the further insights into the mechanistic underpinnings of pain, but assessment and management of pain remains extremely challenging from a clinical management perspective [1].

Because pain is a complex, multidimensional concept that relies most on patient's self-evaluation and psycho-emotional-behavioral reports, it cannot be measured or assessed directly [2]. The naturally subjective nature of pain requires the use of comprehensive practices to accurately assess an individual's degree of pain he/she experiences at the respective moment. The currently main topics of research consider the following objective: to develop an effective objective measure of pain by considering multiple aspects of the pain experience and multiple approaches for miscellaneous types of population and settings.

Recently, despite that the number of studies for objective assessment and effective treatment of pain has grown substantially, the gap between research and clinical application is notable. Clinicians are still currently unable to apply an effective and deterministic approach to assess pain in hospitals and clinical environment that reflects the real level of pain, whose personalized aspects cannot be predicted. Despite continuous attempts to improve the available pain assessment tools and treatments, many patients remain insufficiently relieved [3], while over-dosing is a prevalence in routine environment, i.e. the 'rather too much than too little' concept.

According with recent studies, pain is identified by the American Pain Society (APS) as the fifth vital indicator in diseases and diagnosis chart along with temperature, blood pressure, pulse and respiration rates [4-7]. Recording the pain intensity as 'the fifth vital sign' aims to increase awareness and utilization of objective pain assessment. Although clinicians have a great fundamental knowledge to determine pain treatment based on the recognized causes and physio-pathological mechanisms of pain, an effective objective measurement is needed to overcome the deficiencies in extracting the level of pain in complex environments such as hospitals (in intensive-care, post-operative, post-anesthesia units). Categorizing pain is defined as an instrument to facilitate pain evaluation and treatment, even if it is not for diagnosis. The common ways to classify pain may overlap, as those could be multidimensional or based on a single dimension of the pain experience.

The concepts of fractional calculus are introduced in this medical field for the first time uniquely by our group and aim to provide natural solutions to the modelling aspects of nociceptor pathway considering physiological details of the human body. The paper presents a conceptual framework able to motivate the choice for the fractional tools in a biomedical context. Moreover, this is substantiated by the obtained experimental results, exhibiting features of fractional order systems. To author's knowledge, such features are firstly reported in this paper.

Objectives Assessment of Pain Levels

Basic and clinical research on pain has provided progressive and continuous advances in the last decades [8], especially with regards to a most adequate definition of pain theory and better understanding of the mechanisms and classification of pain. Focusing on those approaches represents the foundation for a comprehensive assessment of pain and treatments used further in pain management.

Assessment of pain should take into consideration physiological, psychological and environmental factors to accurately indicate the existence of pain. Reliable tools for assessment should ensure patient's experience of a safe, effective and individualized pain management along with an appropriate therapy to a patient's response [9], i.e. ideally a personalized pain management framework.

Existing approaches to the evaluation of pain level include verbal and numeric self-rating scales, behavioral observation scales and physiological responses. Subjective tools performed by healthcare professionals to assess pain are: Verbal Rating Scales (VRSs), Numerical Rating Scales (NRSs), Visual Analog Scales (VASs), Faces pain scales, McGill Pain Questionnaire, Behavioral Pain Scale (BPS), Critical-Care Pain Observation Tool (CPOT).

Despite the great efforts that have been deployed in the last decades in order to find adequate ways to objectively measure pain levels in patients and how anesthetic drugs affect a patient's response, no gold standards exist for the assessment of nociception/anti-nociception balance [4].

Nowadays, a variety of monitoring systems are available to provide an efficient way to assess pain in (quasi)real-time. Some of them have been commercialized during the last decade, although not widely used in hospitals by clinicians due to unreliability. There are several objective technologies for pain assessment in clinical evaluation, but none of them are use in daily practice. Recently increased efforts in developing objective measures of pain have resulted in new methods addressing the issue. Recent works related to bioimpedance investigate the muscle electrical properties in patients with low back pain [10] or chronic neck pain [26]. Evaluation of the musculoskeletal pain (pressure pain threshold on myofascial trigger points) has been observed to be corelated with the electrical impedance of the torso [11]. Also, the diagnosis of muscle-strained acute lower back pain could be potentially done by exploring changes in the electrical properties of muscle tissue. This novel technique is bio-impedance based and has been demonstrated as effective in the assessment of neuromuscular diseases. So, bioimpedance could be associated with the physiological properties of muscles, as different pathologies change the normal ones by inflammation or local swelling.

Proposed Methodology

ANSPEC-PRO device

The entire prototype of ANSPEC-PRO device was created in the Research Group DySC (Dynamical Systems and Control) from Ghent University.

ANSPEC-PRO is a measurement device for continuous monitoring of changes in skin impedance as a function of an applied stimulus. The approach is based on the idea that a pain stimulus can be detected from a change in skin impedance as a function of time and frequency. The alteration of the extracellular fluid matrix composition on the nociceptor pathway facilitates the electro-chemical channel communication. Electrical variability in the electrical carrier throughout the signaling pathway, originated by mechanical nociceptor stimulation, affects the response of the skin related in impedance values. Based on this hypothesis, ANSPEC-PRO device has been developed as a proof of concept. The details have been published in [12].

The excitation signal is a multisine signal with 29 components in the frequency interval 100-1500 Hz, with step interval of 50Hz. This signal is sent to the volunteer and the corresponding output is acquired.

The sampling frequency is 15 kHz. The multisine signal is sent with an amplitude of 0.2mA, still a factor 5 below the maximum allowed for patient safety. The measured signals are filtered for noise prior to apply non-parametric identification methods [13]. Given the input is of sinusoidal type, impedance is a frequency dependent complex. Classical periodogram filtering technique has been applied with no overlapping interval, with windowing function Blackman implemented in Matlab environment [13]. The impedance is then evaluated every minute from online data streaming and plotted against frequency. This is then a frequency response either in complex form (Real and Imaginary Parts), either in Bode plot form (Magnitude and Phase).

The procedure followed for measurements and the preliminary results are presented hereafter. Pain intensity was recorded with ANSPEC-PRO prototype and further analyzed with proposed methodology. The set-up of the device for doing the investigational tests is showed in Fig. 1.

Fig. 1. A snapshot of the non-invasive ANSPEC-PRO device for investigational tests

With the novel device available, measurements have been performed on healthy adults. For the measurements, three electrodes were placed on the palmar side of the hand: two current-carrying electrodes (white, red) and one pick-up electrode (black). All measurements were made in a single laboratory, using the same equipment under identical experimental conditions for 8 different volunteers under the supervision of the professor. All authors have followed the official guidelines of good clinical practice.

Mathematical Model

For real-life applicability and engineering approach, the created and validated fractional model is important to create an engineering background for pain evaluation. The results are model-based simulated and evaluated in order to best describe the physiological pain processes with minimally parametrization, flexibility, easily use and computation.

The model has been firstly introduced in [12]. Application of fractional calculus in biology and medicine has shown good characterization of complex phenomena with a startling simplicity, rising great interest in the latest trends [14-18]. In this paper is examined how the concept of a fractional dimension can be applied to time series resulting from physiological processes. The dynamical activity we observe in the natural pain process is related from one level to the next by means of a scaling relation.

The physiological molecular phenomena involved in path transmission is illustrated in Figure 2. This include the multi-scale physiological stages during pain process.

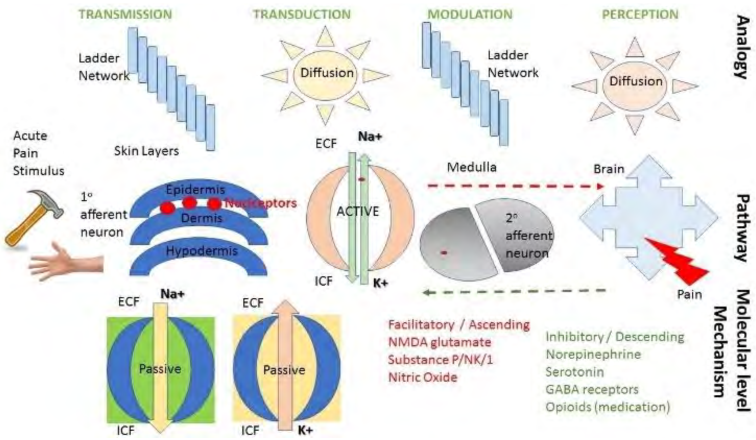

Fig. 2 Analogy of the molecular changes to electrical ladder networks and diffusion mechanisms for pain pathway in healthy individual without drug treatment effects.

The nociceptor pathway is characterized in terms of four processes: transduction, transmission, perception, modulation. Each of this segment of pathway is equivalent to a segment in electrical analogue and diffusion mechanism. This is the top layer of Figure 2. Consequently, following the mathematical convergence and using prior know-how on diffusion approximation terms, we have provided a lumped parameterization of this multi-scale model. Hence, the following Fractional Order Impedance Model (FOIM) is proposed.

$$Z_{FOIM(s)} = R + \frac{S}{s^{\alpha_1}} + \frac{D}{s^{\alpha_2}} + Ms^{\alpha_3} \quad (1)$$

where $0 < \alpha_{1,2,3} < 1$ and R, D, S, M are real numbers. R is a calibration factor, S denotes transmission, D denotes transduction, and M denotes modulation and perception [12].

Notice that not all terms in this model are necessary significant at all times, as some of the physiological processes may be impaired in some conditions (e.g. analgesia will put zero the effect of the modulation and perception term in M). The units are arbitrary, as the model is defined as a difference to the initial state of the patient - due to the use of fractional derivatives - and not as absolute values. This enables patient specificity since no generic model is assumed to be valid and thus broadcasts a new light upon the interpretation of such models. This model uses the fewest parameters that need to be estimated – i.e. seven parameters, while being a personalized model of the individual.

Results

The method and mathematical model presented based on elements of fractional calculus are uniquely defined for each individual studied, analyzing the skin bioimpedance as an indication of absence/presence of nociception.

Every 60 sec, the impedance is calculated and plotted against frequency, by means of its real and imaginary parts. The complex impedance is then normalized and analyzed per interval of pain (P) or no pain (NP), as illustrating the response of the nociceptor excitations.

Impedance of one individual is depicted in Figure 3. It is observed that the lines that denote the first (P1) and second (P2) pain interval responses are very close to the corresponding non-pain intervals: NP3 and NP4. This suggests that NP3 and NP4 indicate the presence of pain latency (i.e. memory pain). However, the interval P3 overlaps with NP1 and NP2, suggesting either of the two possibilities: i) the mechanical pressure was lower, or ii) the ear is less sensitive to nociceptor stimulation. Hence, even in absence of nociceptor stimulation, the impedance indicates presence of pain pathways because of the pain memory effect.

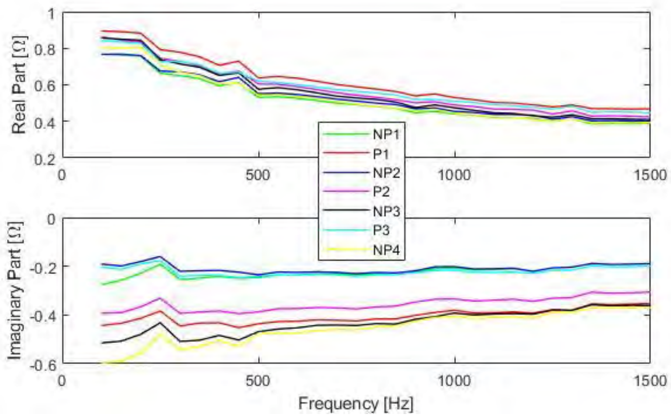

Fig. 3. Individual frequency response of the normalized impedance,
evaluated for “pain”/”no pain” intervals .

Studies have demonstrated that the impedance increases with vasoconstriction and fluctuations in the blood flow determined by the cold water [35]. Same studies show that cold-water test stimulates also the nociceptors, which in term causes sympathetic activation. Thus, vasoconstriction appears also in areas that are not directly exposed to cold and bioimpedance is influenced.

An iterative identification procedure has been performed in order to fit the model to the measured data. The result of the nonlinear least squares identification method applied iteratively for estimating the fractional order impedance model (FOIM) values, applied on a random measurement obtained in this research, is given in Figure 4 for one test interval. The fitting was obtained for the same individual, using the (1) FOIM model. More details regarding the identification method can be found in [12]. The results of following impedance values in all 8 volunteers are depicted in Figure 5. As expected, the biological tissue exhibits a well-known feature of the fractional order systems, i.e. the phase constancy. Indeed, in [18] it has been shown that neuronal ladder network electrical model leads to phase constancy. In nociceptor pathway, this is clearly present as a dominant part of the multi-scale phenomena taking place while stimulus is applied. The latency observed is yet a subsequent feature of such distributed systems.

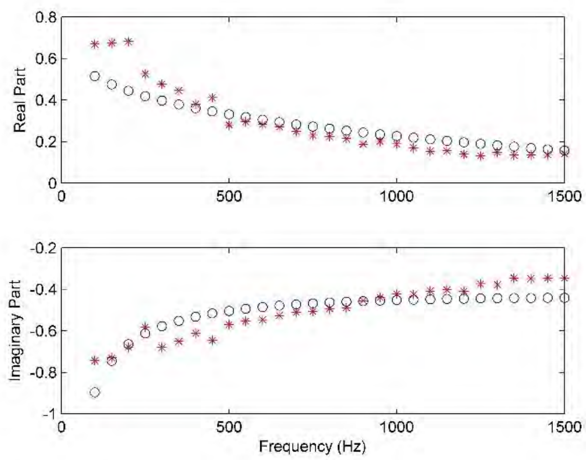

Fig. 4. FOIM results for one volunteer, using protocol #1. Impedance in its complex representation.

Analysis for volume and length of neuronal networks are not available in mathematical form of parametric models, but this may not be relevant in the context of pain assessment. As we are not interested to locate pain origin, this information may not be necessary. The sole purpose is to evaluate the presence and level of experienced pain as to aid in the decision process of pain management and drug therapies.

This last step in correlating the existence of pain with various levels of pain has not been assessed in this paper as all applied stimuli had same intensity. This will be further investigated, taking into consideration a variety of patients types measured during future clinical trials, with different biometric and clinical data.

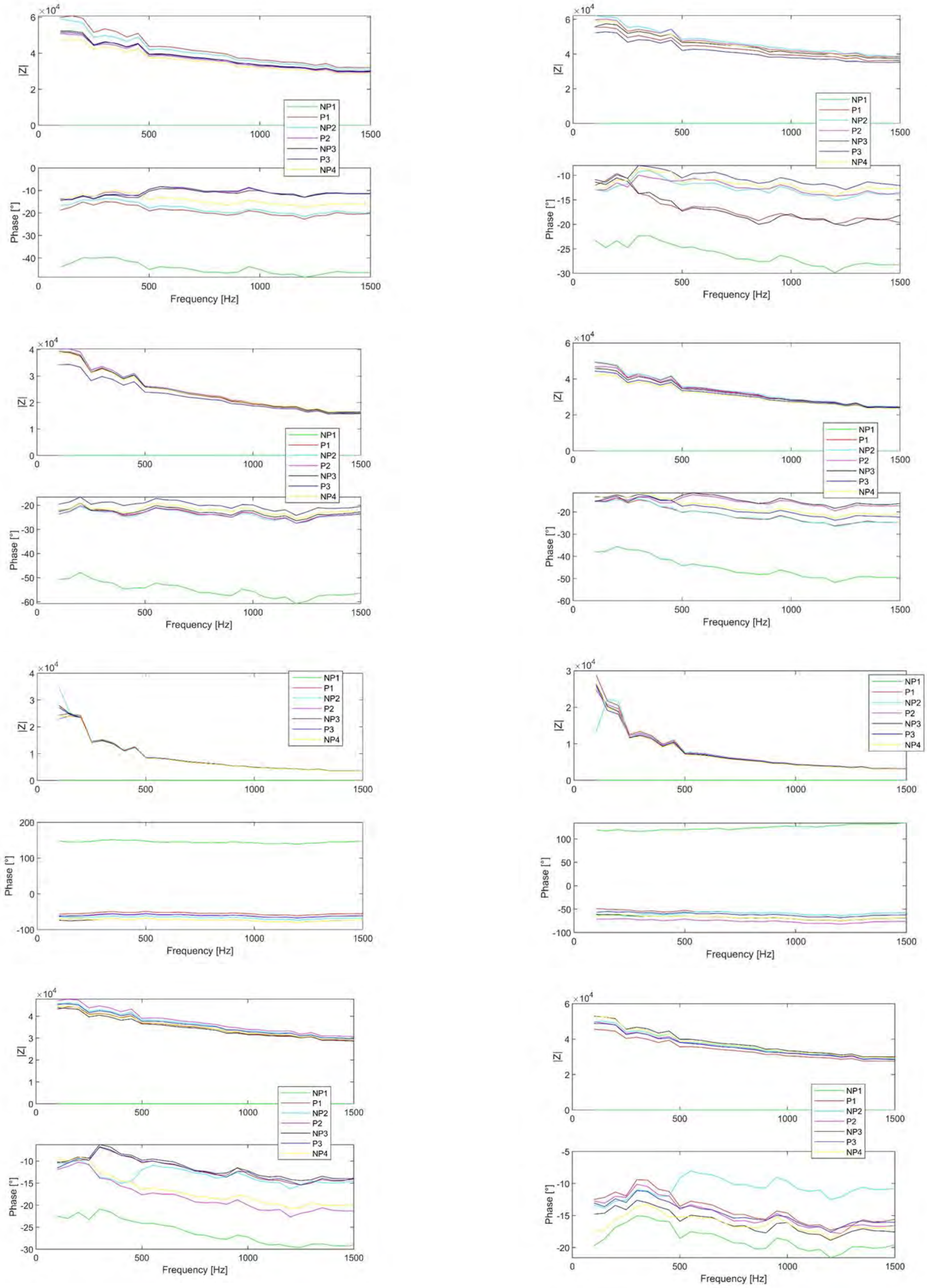

Fig. 5. Absolute values of the impedance |Z|, evaluated for “pain”/”no pain” intervals.

Conclusions and Future Work

This paper presents the first steps to prove the hypothesis that the developed prototype ANSPEC-PRO and the proposed methodology can differentiate between pain and no pain states. From the preliminary results presented here, it follows that the main hypothesis evaluated for mechanical and cold pressures test has been validated. The next step is to correlate the existence of pain with various level of stimulation as to determine a relation or an index suitable for reporting pain levels.

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Fractional-order models of the human respiratory system

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Introduction

The emerging concepts of fractional calculus (FC) in bi- ology and medicine have shown a great deal of success, explaining complex phenomena with a startling simplicity [1,2]. It is clear that a major contribution of the concept of FC has been and remains still in the field of biology and medicine [3]. (Fractional calculus generously al- lows integrals and derivatives to have any order, hence the generalization of the term *fractional-order* to that of *general-order*. Of all applications in biology, linear viscoelasticity is certainly the most popular field, for their ability to model hereditary phenomena with long memory [4]. Viscoelasticity has been shown to be the origin of the appearance of FO models in polymers [5] and resembling biological tissues [6,7].

Viscoelasticity of the lungs is characterized by compliance, expressed as the volume increase in the lungs for each unit increase in alveolar pressure or for each unit decrease of pleural pressure. The most common representation of the compliance is given by the pressure-volume (PV) loops. Changes in elastic recoil (more, or less, stiffness) will affect these pressure-volume relationships. These changes are fueled by structural variations during the progress of age, or pathology, or both. In clinical terms, this is known as *airway remodelling*.

The term *airway remodeling* refers to the process of modification and sustained disruption of structural cells and tissues leading to a new airway-wall structure with implicit new functions. Airway remodeling is supposed to be a consequence of long-term airway diseases. Some studies suggest that the remodeling may be a part of the primary pathology rather than simply a result of chronic inflammation [4]. Of crucial importance in this quest to understand airway remodeling is the composition and structure of the lung tissue [8,9]. The composition and structure determines the mechanical properties of the lungs. Structural changes will induce alternations in tissue elasticity and viscosity.

This task aims to determine a correlation between the structural changes occurring in the lungs and variation in the fractional order value of an impedance model (FOIM). The next section will provide a brief clinical perspective of airway remodelling. The third section presents the electrical analogy to airway models and discusses the effect on the model parameters due to the changes in morphology. The fourth section presents the simulation study of these changes and their effects on the fractional order value. A conclusion section summarizes the main outcome of this work.

Structural Changes in the Lungs with Disease

If a failure in the nominal operation and function occurs in the lungs, the adaptation mechanism will be triggered in an attempt to ensure species survival. This implies ensuring a minimum of vital capacity, which is a balance between changes in alveolar pressure and lung volume during the breathing process. As a defensive mechanism to external agents, the airways and the parenchymal tissue may undergo inflammation, constriction, fibrosis, etc. Structural alternations introduced by pathological processes are traditionally divided into three layers: the inner wall, the outer wall and the smooth-muscle layer. The inner wall exist of the epithelium, basement membrane and submucosa, while the outer layer consists of cartilage and loose connective tissue between the muscle layer and the surrounding lung parenchyma.

In COPD (chronic obstructive pulmonary disease), major structural alternations occur in the small bronchi and membranous bronchiole (airway diameter< 2 mm). Changes occur around the supporting cartilage and bronchial glands in the peripheral airways (2 mm diameter). Here, the thickening occurs mainly in the inner wall area of the large airways [4,9].

The most important changes in asthma are located in the conducting airways (i.e. levels 1-16), which can thicken up to 300%. Asthma patients have thickened segmental and subsegmental bronchial walls over their entire size rage. This thickening is dependent on the degree of the disease, more severe and older patients will depict these characteristics more than young patients [4,10]. In asthma, the inflammatory reactions take place in the higher part of the airways than in COPD. Unfortunately for COPD patients, the airway obstruction that accompanies these changes is resistant to medication which makes the changes persistent. By contrast, in asthma the inflammatory processes can be controlled by the use of corticosteroids. There are also important differences in the remodeling of the extracellular matrix and the role of proteolytic enzymes and growth factors which lead to specific airway remodeling results by disease. More clinical information about inflammation mechanics in airway remodeling can be found in (Berg (2002)). For remodelling effects in asthma, an important role is played by the degree to which the smooth muscle surrounds the airway lumen.

Once an alveolar wall starts to rupture in COPD, the stress the original wall carried is redistributed to the neighboring walls. If this stress is high, a single rupture will induce a cascade of ruptures and serves as a positive feedback for further breakdown. It is obvious that there is a point beyond which the structure-function relationship cannot return to the healthy condition. It is therefore useful to correlate these changes with model parameters for analysis.

Analysis

By analogy to electrical networks, one may consider voltage as the equivalent for respiratory pressure P and current as the equivalent for air-flow Q. Electrical resistances Re may be used to represent respiratory resistance that occur as a result of air-flow friction in the airways. Similarly, electrical capacitors Ce may represent volume compliance of the airways which allows them to inflate/deflate. We will discuss in this section their definitions as a function of morphology and their role within the airway remodelling process.

Parameters

In [11] was developed the electrical analogy to transmission lines for the mechanical parameters in the elastic and viscoelastic airways. The model was based on the geometrical parameters: radius and wall thickness on the mechanical characteristics of the airway tube: complex elastic moduli (given by its modulus and angle) and Poisson coefficient and on the air properties: viscosity and density. A detailed mathematical explanation in given in [12]. The values for airway radius, thickness, elastic moduli, etc. have been used as those published in [12]. The resistance and compliance per branch are depicted in Fig. 1 left. The consecutive ratios between these branches for resistance and compliance are given in Fig. 1 right. It can be observed that the ratios of resistance are above 1, indicating an increase with every level - this is obvious, since we are looking at each branch solely, and the cross-sectional area is diminishing. The compliance is below 1, indicating that it increases, as a result of the elasti moduli, with more soft tissue and less cartilage percentage at lower branches in the respiratory tree. This is indeed the case, since these lower airways are those which need to expand during inspiration (i.e. alveolar levels where diffusion occurs).

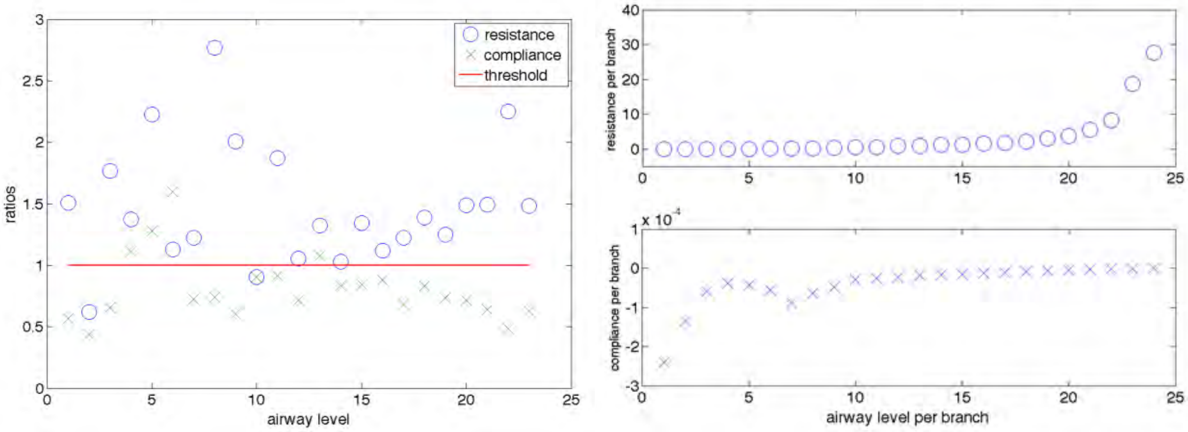

Fig. 1: Evolution of resistance and compliance values per branch in normal airways - left; Evolution of the ratios in normal airways with consecutive branches in consecutive airway levels - right.

Changes in the airway radius, wall thickness and elastic modulus

Here we investigate the effect of changes in the airway radius, airway wall thickness and elastic modulus of the tissue. These changes are significant for obstructive disease such as asthma and COPD. However, the most important are the irreversible changes in COPD, since these will always mark the respiratory impedance and be visible in all lung function tests. Changes in the respiratory zone (i.e. aiways below level 16) in the distal airways and lung parenchyma. First, let us look at changes in the airway thickness. This may occur since the lung begins to 'protect' itself from the damaged cells in the airway soft tissue. In time, fibrosis occurs and thickness may increase significantly. Figure 2 left depicts the variation in the resistance and compliance ratios per branch with each airway level in case of increased changes in thickness with 50%. Second, the change in thickness usually induces a change in the radius. This become obstructed and mucus may result from the inflammation of the airway soft tissue. Mucus secretion may obturate totally parts of the airway, such that produces exacerbations and the patient will cough with sputum secretions. Figure 2 right depicts the variation in the resistance and compliance ratios per branch with each airway level in case of decreasing the radius with 50%.

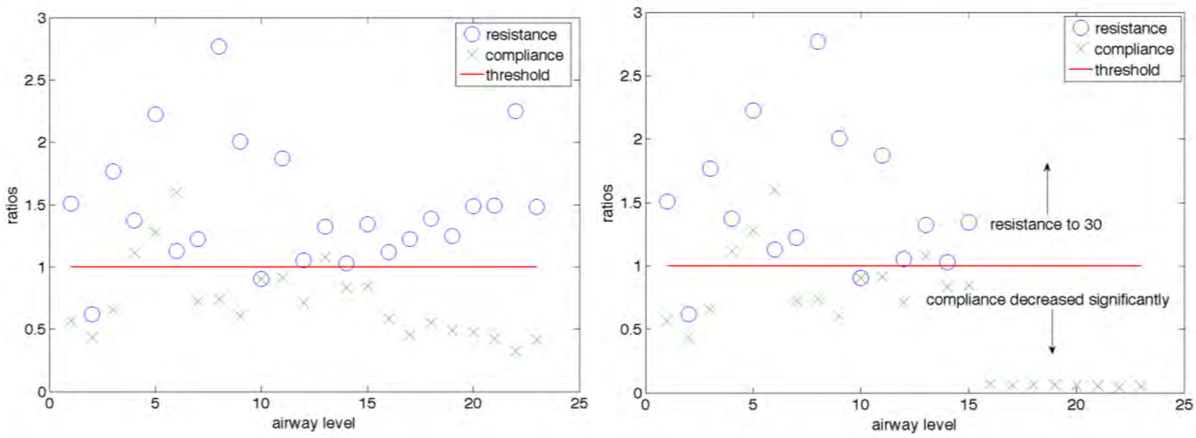

Fig. 2: Evolution of the ratios in thickened airways with consecutive branches in consecutive airway levels - left; Evolution of the ratios in thickened and obstructed airways with consecutive branches in consecutive airway levels – right

Finally, the changes in thickness and radius are usually resulting in a change in the elastic modulus of the soft tissue and the cartilage tissue in the airways. Typically, fibrosis will reduce the overall elasticity of the modulus. Figure 3 depicts the variation in the resistance and compliance ratios per branch with each airway level in case of stiffening the soft and cartilage tissue with 30%. It is obvious that the changes with most impact are those related to obstruction. Fibrosis, a slow process, will also contribute to the increase in resistance and thus will impede the air passage through the airways. The wall ruptures discussed in Figure 1 will eventually decrease the resistance because the air will be able to pass easier through.

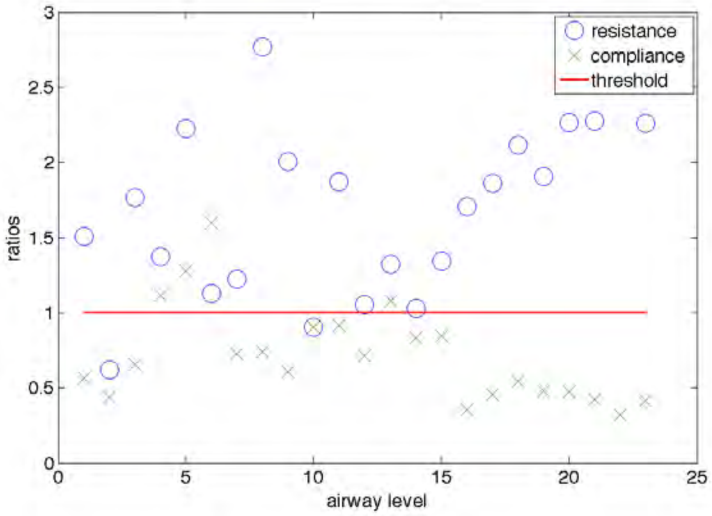

Fig. 3Evolution of the ratios in fibrotic airways with consecutive branches in consecutive airway levels.

Results

The respiratory tract can be approximated well by its electrical analogue, where current denotes flow changes and voltage denotes pressure changes. If the morphological structure of the lungs is preserved, then the quasi-fractal structure of the lungs may be employed, simplifying significantly the mathematical burden of the model. It has been shown in [12] that a ladder network with recurrent impedance elements. In the frequency domain, the fractional order will lead to a constant-phase behavior, i.e. a phase locking in the frequency range given by the convergence conditions [12]. Depending on the number of cells in the ladder (N), the constant phase behavior will emerge over a wider range of frequencies. This result is applicable to any kind of ladder network (airways, arteries, etc.). However, the fractional order value and coefficients will change according to the properties (morphology, geometry) of the system. For the changes discussed in the previous section, one may calculate the corresponding variations in the fractional order value n. Figure 4 shows the variations occurring in the magnitude and phase of the impedance with increasing values in the ratios of the resistance.

The effect of decreasing compliance (i.e. increasing stiffness) will be opposite to that of increasing resistance. However, is the relative degree of changes between the two which dictates the increase or decrease of the fractional order parameter with the progress of the respiratory disease.

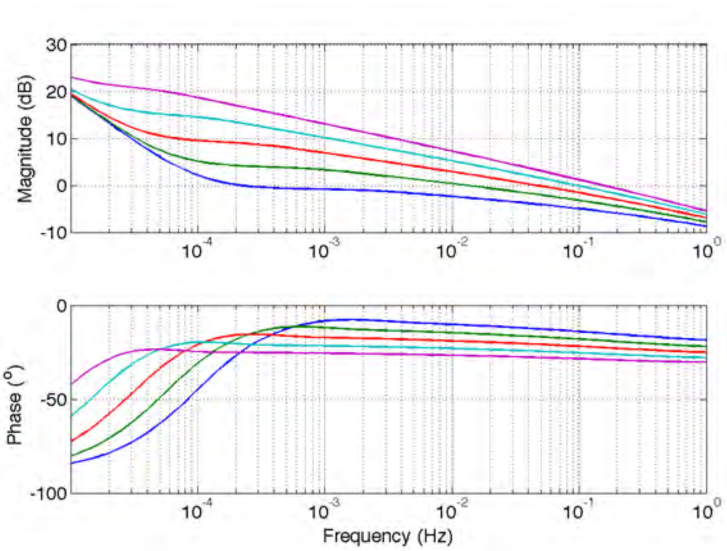

Fig. 4 Evolution of the impedance with increasing values of resistance ratios

Conclusions

It is straightforward to apply airway altering/remodeling effects in the simple model representation proposed here, but limitations should be taken into account. The major errors which may occur in this study are determined by the heterogeneity of the human lung, i.e. inter-subject variability can affect the morphologic values of airway radius, thickness, length and tissue elasticity. Another further simplification in our reasoning is considering negligible the effects from the branching angles. These angles influence the flow to change direction, may lead to an asymmetrical velocity profile, to develop a secondary flow in the daughter branches and the inner airway walls to be slightly stretched. The change in cross-sectional areas which occurs from parent to daughter branches in a bifurcation causes the fluid to undergo a deceleration and may cause separation of adjoining streamlines. However, this kind of information may be more useful to study airflow dynamics in aerosol deposition models rather than in lumped impedance models.

A correlation between the structural changes occurring in the lungs and the corresponding variations in the fractional order value of an impedance model was provided. Discussion on variations in the wall thickness, cross-sectional area and elastic moduli in distal airways involved in respiratory process affect changes in the fractional order value have been also given. Two lumped models were discussed: i) a theoretical model derived from morphological information and ii) an identified lumped parametric model. The relationship between fractional order term and heterogeneity in the lungs has been related to changes in viscoelasticity. Our results indicate that a correlating analysis is possible for various degrees of obstruction and effects may be directly related to the identified fractional order value.

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FO modelling and control of swimming microrobot actuated by smart materials

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Introduction

The emerging targeted drug delivery concept is an area of the nanomedical sphere focused on applying localized treatment to isolated areas. Many advantages distinguish between faster action with less administered substance and reduced side effects. Instead of tainting the body with an increased amount of substance spreading throughout the entire anatomical frame, the targeted approach aims at fathoming the problem in a secluded manner [1], [2]. The novelty of the present study lies in bridging the nanomedical and the applied control engineering fields by tackling the issue of the carrier nanorobot able to deliver the needed treatment in the focused areas. The carrier unit is upsized to a scalable robot with autonomous submerged capabilities used to determine and experimentally validate the interaction between the carrier and the vascular environment, while also providing a controlled movement in terms of velocity and position. The dynamics of the carrier robot are modeled using a novel approach on accurately modeling physical phenomena lying in the fractional calculus domain [3]. Fractional calculus provides better understanding of the surrounding world by eliminating the limitations procured by integer order differentiation [4] proving superiority in modeling of viscoelastic characteristics [5], [6].

Controlling the obtained complex fractional order model of the dynamics of the robot shouldn't be limited to classical integer order controllers. The classical Proportional Integral Derivative (PID) control is a particular case of the extended, more flexible and complex fractional order control perspective. Fractional order controllers prove superiority in terms of obtained performance both in the transitory and steady-state regimes by providing more complex control with increased degrees of freedom, capable of honoring a larger number of performance specifications for the chosen process [7]-[9].

Experimental Platform

The experimental platform has been designed and built at the Technical University of Cluj-Napoca in collaboration with Ghent University. The main components distinguish between the vascular platform, scalable submersible and the concentration measurement unit. The submersible is inspired from the field of Automated Underwater Vehicles (AUV) [10] in terms of operability and physical characteristics with additional biomedical singularities. The robot has been designed such that it's ellipsoidal shape reduces the drag effect while moving in a submersed environment. The submersible is equipped with a single Graupner 40mm3 blades propeller that acts as the thrusting and braking element of the system. The autonomous characteristic of the robot is given by embedded electronics consisting of a WI-FI module and microcontroller ESP8266 that registers acceleration data from the 9-DOF BNO055 IMU unit and controllers the propeller's movement by sending PWM signals to a geared DC motor. The concentration is measured using the DropSens DS550 which is a screen printed platinum electrode capable of measuring the glucose concentration of a liquid. The measured concentration and its position are registered and sent via the Wi-Fi unit to an external server that logs the data for possible further analysis.

Also, the external server is capable of overriding the submersible's autonomous movement through external PWM values, used just as a security assurance in case of emergencies.

The robot described previously is introduced through the immersion point, travels through the tube resembling the vein and passes a vascular transition inside the artery. Ultimately, the robot is extricated through the extraction container. In a single test, the robot is capable of analyzing the concentration of the liquid at different positions. For the purpose of this study, the exact concentration values are irrelevant, only the abrupt changes are of interest. An experimental test is presented in Figure 1. At t = 20 seconds, the concentration value drops from the original value to a much lower value. This has been realized experimentally by introducing a mixture of sugar and warm water inside the artery shortly before the robot's pass through that area. The DS550 sensor, detects the change and the microcontroller logs the relevant data which is sent to the server.

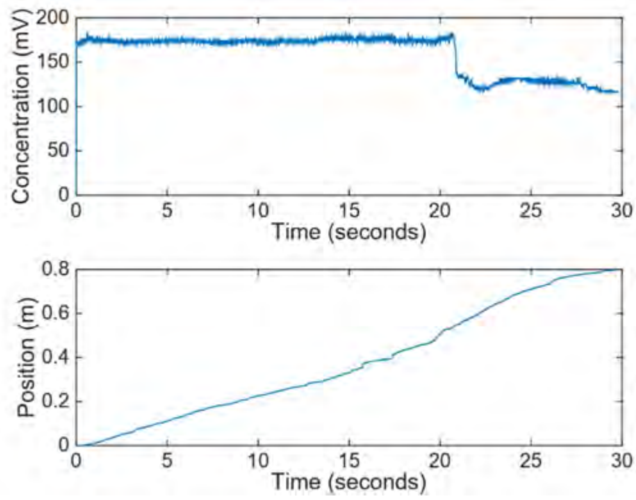

Fig. 1 Concentration and position data registered by the submersible in the circulatory system

The targeted drug delivery concept [13] is applied to the system by offering a solution to treat the high concentration area in the following manner: the robot travels through the circulatory system until a blunt change in the concentration data is perceived identifying the area in need of localized treatment; the robot stops in the area of interest in order to perform the substance administration; when the concentration drops to normal levels, the robot continues it's path through the circulatory system repeating the process. The present work focuses on stopping the submersible in the targeted area, while the drug administration application and dosage is ignored. Hence, the focal point of the study is to control the velocity and position of the robot in order to ensure fast and accurate action in the areas in need of treatment.

Fractional-order Model and Control Tuning

The motivation of searching for a fractional order model to characterize the dynamics of the submersible's movement lies in the viscoelastic characteristic of the blood vessels disseminated through expansion and contraction. The dynamics of the submersible's movement consist of expressing the movement along the longitudinal axis as a relation between the applied PWM to the propeller and the velocity/position of the robot. Since the position is the first order derivative of the velocity and a fractional order model is desired, the velocity profile is approximated through a fractional order transfer function resembling a second order model.

The experimental fractional order identification consists of imposing the shape of the fractional model and optimizing the parameters such that the integral absolute error with respect to the experimental data is minimized. Initial conditions are provided for the fractional orders, the static gain as well as the coefficients. The minimization is performed using the fmincon function provided by Matlab's Optimization Toolbox. The optimization algorithm searches for a solution to minimize the absolute error between measured position data and the response of a possible candidate model. The test has been performed by giving a PWM duty ratio of 0.3 in a fixed period of 10 Hz. The obtained position model compared to the experimental data is presented in Figure 2. As can be seen, the model is successfully validated on the experimental data.

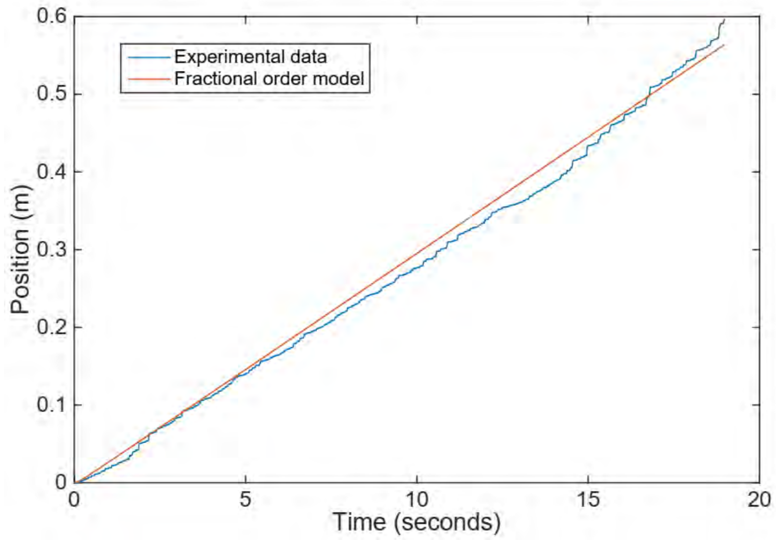

Fig. 2: Identified fractional order position model validation on experimental data

The main purpose of the controller is to travel with a specified velocity and to stop the submersible's motion in a desired area. In order to do so, the velocity should drop to 0 as fast as the area is located and keep its position regardless of any disturbance. If the absolute velocity of the submersible is 0 with respect to the liquid's movement, the position of the robot is constant. The

proposed control strategy is applied to the fractional order velocity model in order to stifle the movement of the robot with 0 steady state error, ensuring effective and accurate drug administration at the aimed spot. For this particular requirement, a fractional order Proportional Integrative (FOPI) controller is chosen. Several fractional order controller tuning procedures can be applied to determine the three parameters of the desired controller: kp, ki and lambda, such as the ones described in [7, 9]. The chosen control strategy is based upon frequency domain constraints imposed by taking into consideration the targeted drug delivery process' requirements.

Two design constraints are associated to a stable closed loop system with reduced settling time. This is realized by imposing the gain crossover frequency of the magnitude Bode plot and the phase margin of the open loop system with the FOPI controller. The third constraint is imposed such that the obtained controller overcomes particular traits of the circulatory system of every individual, requiring a certain degree of closed loop system robustness. The particularities of the blood flow are given by the ability of blood vessels to expand and contract, different blood viscosities or different blood flow profiles as a result of certain diseases or advanced aging [14, 15]. The robustness attribute is mathematically described by a constant phase in an interval comprising the gain crossover frequency. By imposing the open loop phase derivative as being 0, the phase margin of the system remains constant for certain process gain uncertainties, ensuring robust performance to small gain changes.

Results

A fractional order PI controller is determined based on the tuning procedure presented in the previous subsection. When choosing the desired phase margin and the gain crossover several rules should be respected such that the obtained parameters have physical meaning [16]. The imposed phase margin is 65o, while the gain crossover frequency is chosen as 6 rad/s. Solving the equations related to the magnitude, phase and phase derivative gives the FOPI velocity controller. The frequency response diagram proves that the imposed constraints are honored in Figure 3 left. The controller is validated by its ability to reduce the velocity of the submersible to 0 in the arterial environment and restoring the velocity to the operating velocity after the area has been treated. Figure 3 right presents the behavior of the closed loop system with the computed controller overlaid on the uncompensated system behavior. Between 0 and 1 seconds, the velocity is restored from 0 m/s to 0.05 m/s. The efficacy of the controller can be clearly seen compared to the uncompensated system, especially in the case of the steady state error. For the time period between 1 second and 2 seconds, the control action acts as a breaking unit for the submersible.

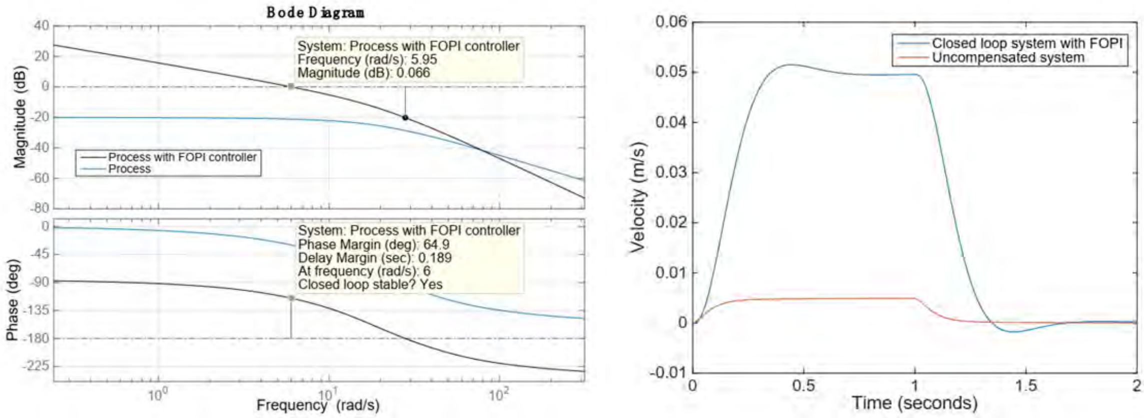

Fig. 3: Frequency response diagram of the closed loop system with FOPI controller – left; Velocity profile of the uncompensated process compared to the closed loop system – right

The position evolution is presented in Figure 4 left. Between 0 and 1 seconds, the velocity of the robot settles around 0.05 m/s, causing a gradual position change. At $t = 0.5$ seconds, the velocity drops to 0 in order to obtain a fixed position for the submersible. The command signal computed by the FOPI controller is the PWM duty ratio applied to the motor with a fixed frequency of 10 Hz. The computed signal is plotted in Figure 4 right. The braking effect is visible at t = 1 second by rotating the propeller in the opposite direction. The command signal is limited between [-1,1] because of the PWM duty ratio. The simulated behavior of the closed loop system proves successful in improving the response of the submersible's velocity and position, validating the obtained fractional order controller.

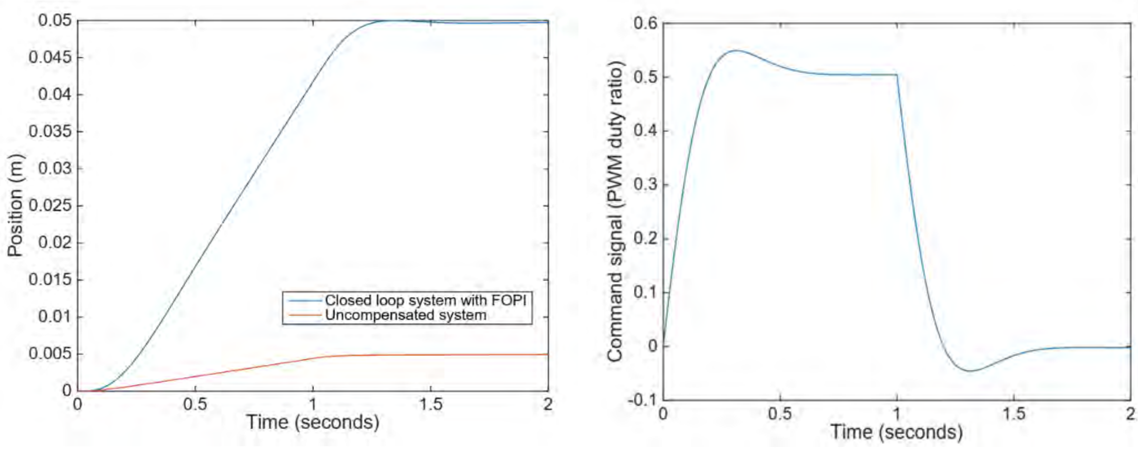

Fig. 4: Position of the uncompensated process compared to the closed loop system - left; Command signal computed with the FOPI controller – right.

Conclusion

The paper presents the modeling and control of a scalable robot fit for the targeted drug delivery field. The experimental setup resembling the circulatory system allows measurements of the fluid's concentration and the submersible's position. The interaction between the underwater robot and the surrounding environment in terms of velocity and position profiles is modeled using a complex fractional order model motivated by the viscoelastic characteristic of the blood vessels. For the obtained model, a fractional order PI controller is determined in order to improve the velocity and positioning of the robot with the purpose of delivering targeted treatment in the areas with different blood characteristics. The controller's effectiveness is emphasized in terms of steady state error and stability while also having a certain degree of robustness to environmental changes. Future work includes comparison of the proposed tuning approach with other control options for velocity control. Also, a fractional order analytical model for the submersible's position dependent on the non-Newtonian characteristic of the blood flow is going to be determined. The development of position control strategies to ensure that the submersible navigates to a given position in order to apply treatment and stays in the given position regardless of blood flow disturbances is taken into consideration.

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Modelling the behaviour of biological neurons with fractional-order differential equations

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Introduction

In electrophysiological experiments, the neuronal membrane is considered to be equivalent to a resistor-capacitor circuit, and it has been claimed that the dynamic behaviour of neurons can be more appropriately modelled by a non-ideal capacitor and/or inductor [1], in which voltage-current relationship is given by a fractional-order derivative. Several research results concerning biological neurons [2, 3] also suggest that mathematical models of neuronal activity could be improved by using fractional-order derivatives. Moreover, [4] advocates that the index of memory could be a physical interpretation of the order of a fractional derivative, which is in compliance with engaging fractional-order operators in neuroscience modeling.

Fractional-order membrane potential dynamics have confirmed their advantage in reproducing the electrical activity of neurons observed from an experimental point of view, as they are capable to introduce capacitive memory effects [5]. Very recently, [6] proposed a novel mathematical model of neuronal electromechanics employing fractional order derivatives of variable order to model multiple temporal scales, accounting for both local and nonlocal chemomechanical interactions observed experimentally [7].

Several types of fractional-order neuronal models have been investigated in the recent years: leaky integrate-and-fire model [8], Hindmarsh-Rose models [9, 10], Morris-Lecar models [11,12,13,14], FitzHugh-Nagumo model [15,16], Rulkov model [17] and the more general Hodgkin-Huxley model [18]. In this report, we summarize several results which have been obtained regarding this topic in the framework of the COST Action CA15225.

The Morris-Lecar model

Jacques Curie’s empirical law for the current through capacitors and dielectrics leads to the following capacitive current-voltage relationship for a non-ideal capacitor:

$$I_c^\alpha = C_m^\alpha \frac{d^\alpha V_c}{dt^\alpha}$$

where $0 < \alpha < 1$, the fractional-order capacitance with units (amp/volt)sec$^\alpha$ is denoted by C_m^α and d^α/dt^α represents a fractional-order differential operator [5].

Starting from the classical Morris-Lecar neuronal model [19] which describes the oscillatory voltage patterns of Barnacle muscle fibers, in [14] a fractional-order model has been constructed as follows:

$$\begin{cases} C_m(q_1)\cdot {}^c D^{q_1} V(t) = g_{Ca} M_\infty(V)(V_{Ca} - V) + g_K N(V_K - V) + g_L(V_L - V) + I \\ {}^c D^{q_2} N(t) = \overline{\lambda}_N^{g_2} \cdot \lambda(V)(N_\infty(V) - N) \end{cases} \tag{1}$$

where V is the membrane potential, N is the gating variable for K^+, I represents the externally applied injected current, V_{Ca} , V_K and V_L denote the equilibrium potentials for Ca^{2+} , K^+ and the leak current ($V_K < V_L < 0 < V_{Ca}$), g_{Ca} , g_K and g_L are positive constants representing the maximum conductances of the corresponding ionic currents, and $\overline{\lambda}_N$ is the maximum rate constant for the K^+ channel opening. Here, $C_m(q_1) = \tau^{q1}/R_m$ is the membrane capacitance [5], R_m is the membrane resistance, τ is the time constant. The dimensional consistency of this model is guaranteed by including the fractional-order capacitance to the left hand-side of the first equation and of the term $\overline{\lambda}_N^{g_2}$ to the right hand-side of the second equation of system (1).

As previously considered in the literature, the following functions are considered in (1):

$$M_\infty(V) = \frac{1}{2}\left(1 + \tanh\left(\frac{V - V_1}{V_2}\right)\right),\ N_\infty(V) = \frac{1}{2}\left(1 + \tanh\left(\frac{V - V_3}{V_4}\right)\right),\ \lambda(V) = \cosh\left(\frac{V - V_3}{V_4}\right),$$

where V_i are positive constants, $i \in \{1,2,3,4\}$.

Considering $q_2 = 1$, the existence and stability of the equilibrium states of system (1) have been investigated in [14], revealing the co-existence of three branches of equilibria for a certain range of the externally applied current I. It has been also shown that while one of the branches of equilibria is unstable, for any value of the fractional order q_1 , the equilibria belonging to the other two branches might be asymptotically stable for certain values of q_1 and unstable for others. The existence of stable limit cycles has also been obtained, for certain values of I. Numerical simulations revealed that for the same value of I, as the fractional order q_1 decreases, the number of spikes over the same time interval increases, which may correspond to a better reflection of the biological properties by the fractional order model.

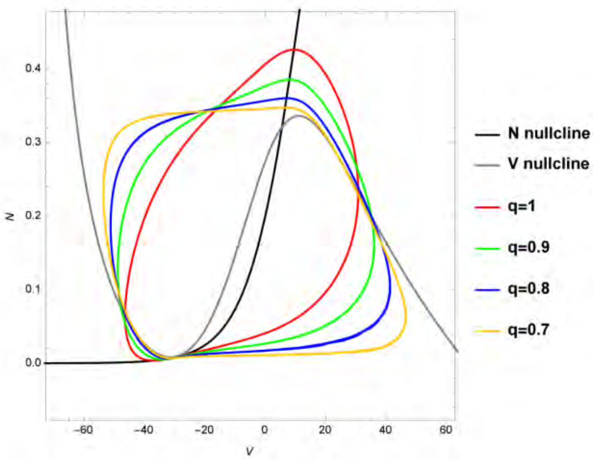

Figure 1: Limit cycles for the Morris-Lecar model (1) with various fractional orders q_1

The Fitzhugh-Nagumo model

An extension of the classical FitzHugh-Nagumo model [20] involving Caputo fractional order derivatives has been investigated in [15]:

$$\begin{cases} {}^cD^{q1}v(t) = v - \dfrac{v^3}{3} - w + I \\ {}^cD^{q2}w(t) = r(v + c - dw) \end{cases} \tag{2}$$

where v represents the membrane potential, w is a recovery variable, I is an externally applied injected current and $0 < q_1 \le q_2 \le 1$. A similar model has been investigated by means of numerical simulations in [21].

The existence of equilibrium states and their stability have been fully analyzed, leading to a full characterization of the set of fractional orders (q_1 ,q_2) for which a certain equilibrium of system (2) is asymptotically stable. Oscillatory and spiking behavior has also been observed, and linked to Hopf-like bifurcations caused by switches of the fractional orders (q_1 ,q_2) from the stable region to the unstable region (see Fig. 2).

A discrete time counterpart of system (2), involving Caputo-type h-difference operators, has also been analyzed in [16], with the following form:

$$\begin{cases} {}^{c}\Delta^{q1} v(nh) = I - I(v(nh), w(nh)) \\ {}^{c}\Delta^{q2} w(nh) = \phi(w_{\infty}(v(nh)) - w(nh)) \end{cases} \tag{3}$$

with $I(v,w) = w - v + \frac{v^3}{3}$ and $w_{\infty}(v) = \alpha v + \beta$ is a linear function. A similar theoretical analysis has been undertaken as for the continuous-time counterpart, revealing the effect of the fractional orders as well as the discretization step size h on the stability of the equilibria and the oscillatory and spiking behavior of system (3).

The Rulkov model

A discrete-time fractional-order Rulkov-type model, describing the spiking behavior of a biological neuron has been investigated in [17]:

$$\begin{cases} {}^{c}\Delta^{q1} x(n) = \dfrac{\alpha}{1 + x(n)^2} - x(n) + y(n) \\ {}^{c}\Delta^{q2} y(n) = -\mu(x(n) - \sigma) \end{cases} \tag{4}$$

where x represents the membrane potential, y is a gating variable, with $0 < \mu \ll 1$, σ acts as an externally injected current applied to the neuron and $\alpha > 0$ is the coefficient of the nonlinear term of the Rulkov neuronal map and $0 < q_1 < q_2 \leq 1$. The same investigation pattern has been followed as for the previously considered model: a theoretical analysis of the equilibria and the characterisation of their stability properties in terms of the fractional orders q_1 and q_2, followed by extensive numerical simulations to exemplify the different dynamic regimes exhibited by the considered model.

Conclusions

In this report, several fractional-order neuron model have been reviewed which have been previously explored in the references [14, 15, 16, 17]. Numerical simulations which have been undertaken suggest that the fractional-order version several well-known neuronal models may provide a more realistic modeling of individual spikes compared to their classical integer-order counterparts. The frequency and amplitude of spikes can be successfully modulated by the fractional orders of the considered fractional-order derivatives.

As a direction for future research, synchronization between these neuron models will also be investigated. A detailed bifurcation analysis will be performed involving bifurcations of limit cycles and it will be examined whether these models may exhibit chaotic behavior under the certain conditions.

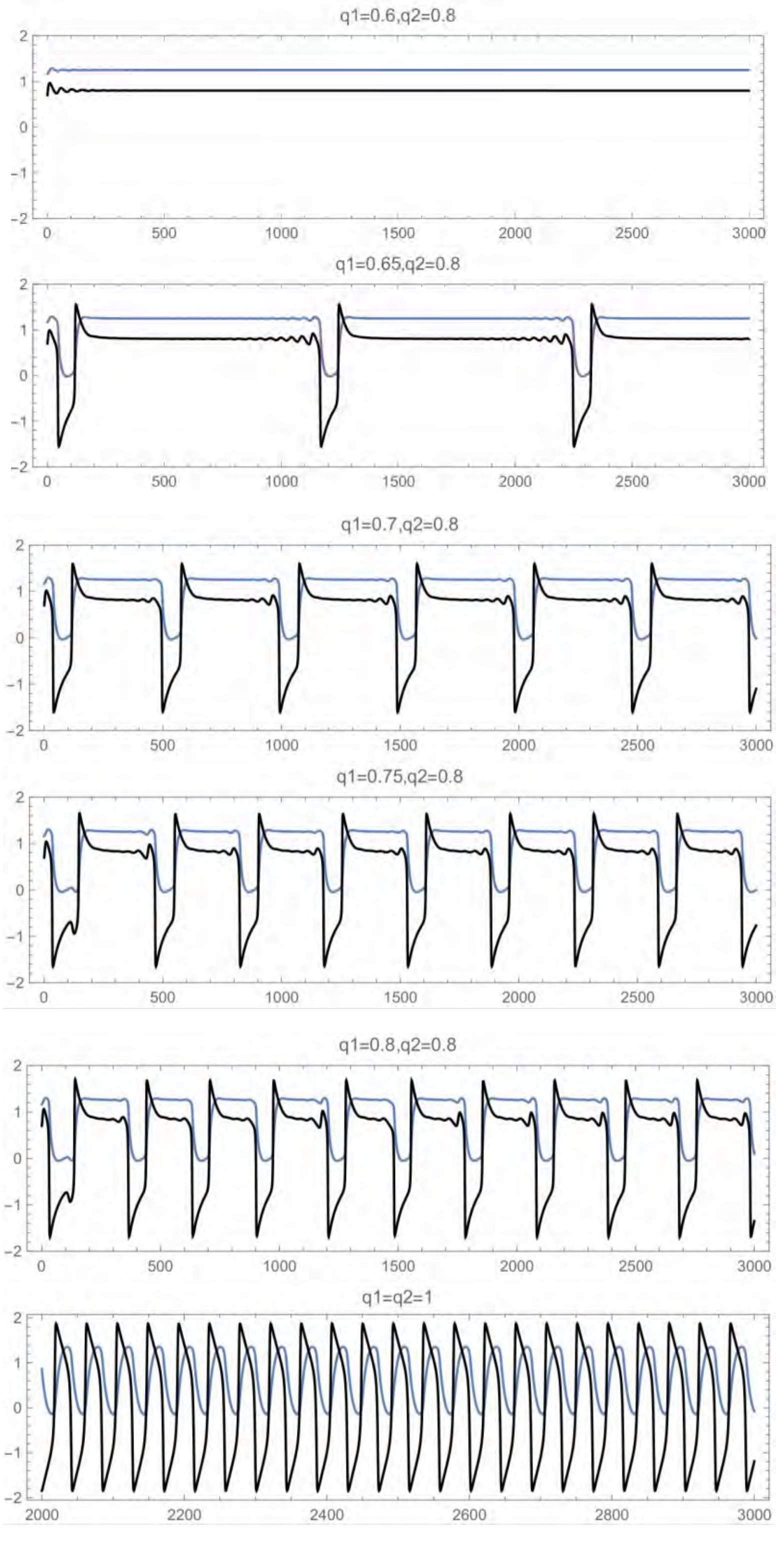

Figure 2: Evolution of the state variables of system (2) (with parameter values: r = 0.08, c = 0.7, d = 1.2 and I = 1.25) for different values of the fractional orders.

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